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On the Management of Self-Owned Containers

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Résumé - Cette étude développe des solutions pour optimiser le repositionnement des conteneurs vides dans l′arrière-pays. Les conteneurs arrivent au fil du temps chez le destinataire (importateur), tandis que la demande de conteneurs provient de l'expéditeur (exportateur). Un conteneur vide peut être loué par l'expéditeur auprès du destinataire pour envoyer sa cargaison au terminal maritime. Nous modélisons le système comme une file d'attente à deux extrémités avec un temps d'appariement non nul et un nombre limité de ressources. Le destinataire cherche à minimiser la somme des coûts de stockage et de déplacement en décidant quels conteneurs doivent etre conservés pour une location future et lesquels doivent etre immédiatement renvoyés au terminal maritime. Par le biais d'un processus de décision de Markov, nous prouvons qu'avec un seul camion, la politique optimale est une politique à seuil dépendant de l′état où les seuils d'admission augmentent avec la quantité stockée chez l′expéditeur et la demande de l′expéditeur, et diminuent avec la distance entre le destinataire et l′expéditeur. L′intensité du trafic joue un rôle de la même ampleur que le ratio d′importation/exportation pour les décisions de retenue. En particulier, la réduction relative des coûts obtenue par la politique de retenue est la plus grande dans les zones à faible trafic, révélant qu'un stock dans l′arrière-pays compense la rareté des conteneurs.

Abstract - This study develops solutions to optimize empty container repositioning in the hinterland. Self-owned containers arrive over time at the (importer) consignee, while the demand for containers arises from the (exporter) shipper. An empty container can be rented by the shipper from the consignee to send its load to the sea terminal. We model the system as a double-ended queue with non-zero matching time and limited number of resources. The consignee aims to minimize the sum of holding and travel costs by deciding which containers should be stored for future rent and which should be immediately returned to the sea terminal. Through a Markov decision process, we prove that with a single truck, the optimal policy is a state-dependent withholding threshold policy where the admission thresholds increase with the quantity stored at the shipper and demand by the shipper, and decrease with the distance between consignee and shipper. Traffic intensity plays a role of the same magnitude as the import/export ratio for withholding decisions. Specifically, the relative cost reduction obtained by the withholding policy is greatest in low-traffic areas, revealing that an inventory in the hinterland compensates for the scarcity of containers.

Mots clés - Conteneur, file d′attente à double extrémité, stock, Processus de décision Markovien, tournée de rue.

Keywords - Container, double-ended queue, inventory, Markov decision process, street turn.

1 INTRODUCTION

Containerization is a standardized system of freight transport that moves containers from door to door. This includes container ships, deep sea terminals with special handling equipment, and intermodal infrastructure in the hinterland such as inland terminals. The United Nations Conference on Trade and Development stated that in 2017 around 80% of global trade by volume or 70% by value was carried by sea and handled by ports (UNCTD, 2018). In line with the growth in intercontinental maritime transport, hinterland container traffic has grown substantially. In particular, transportation modes such as barges, trains, and trucks have been adapted to transport containers to and from deep sea terminals.

Once a container has been unloaded at its destination in the hinterland, another transport leg must be found, as moving an empty container is almost as costly as moving a full container. Moreover, whether or not it is loaded, a container takes up the same amount of space and thus requires the same transport capacity (Notteboom and Rodrigue, 2021). In an ideal situation, an inbound container should find an outbound load once it has been unloaded before being sent back to the sea terminal. The strategy of matching an empty container from an importer with a load from an exporter, so that the container is full in both directions, is called a *street turn strategy*. In practice, however, containers are often immediately sent back empty from the hinterland to the sea terminal, leading to additional transportation costs and pollution. An important reason for these empty movements is the imbalance between imports and exports. However, other factors incentivize the immediate return of empty containers to the sea terminal. In particular, distance and a lack of coordination between importers and exporters in the hinterland may discourage operating matches between empty containers and outbound loads. More importantly, the high detention costs imposed by shipping lines create an urgency to send back empty containers instead of storing them until a match can be found.

Using data from 2017, Legros et al. (2019) revealed that the cost savings obtained by a street turn strategy were limited, although a high proportion of containers could be reused. Recently, the motivation to implement a street turn policy has declined even further. With the rapid increase in global container shipments from late 2020 through 2022, a global shortage of containers has been reported (e.g., Bhattarai, 2021). As a result, ocean carriers have increased their demurrage and detention fees and have limited free detention periods (Angell, 2021). Specifically, xChange (2022) reported that the global average of demurrage and detention charges increased from \$586 in 2020 to \$664 in 2022, corresponding to an increase of 12%. Detention costs vary significantly from one region to another; they were around \$2,500 at day 14 in Los Angeles in 2022 for a standard 40ft, while they are around only \$120 in Busan. In areas where detention costs are highest like New York or Los Angeles, we observe that they reach the same cost as buying a container that can be reused several times, which is around \$3,900 for a standard 40ft. These conditions almost entirely preclude the implementation of a street turn policy.

Conversely, legal restrictions on carbon emission and recent increases in fuel prices amplify the need to reduce empty container movement by reusing them for export operations. Avoiding detention fees and reducing travel costs are thus contradictory drivers. In the current situation, the high detention fees are preponderant in decision making, which means that most containers are immediately returned to the sea terminal. A simple way to avoid detention fees, obtain access to and control over containers, and implement a street turn strategy is to use Self-Owned Containers (SOC) instead of Carrier-Owned Containers (COC). In the current context, SOC management is becoming an increasingly important issue for many companies (Shores, 2021). With SOCs, an importer can decide to rent its own containers to an exporter to avoid empty trips from the hinterland to the sea terminal, thus implementing the street turn strategy. Although SOCs allow importers to make decisions more independently, they require careful management because some SOCs should be stored for future rentals at the importer's location, while some should immediately be returned to the sea terminal when the imbalance between imports and exports is high or when finding a match will take too long.

In the present study, we examine the optimization of container storage decisions by an importer in the hinterland while including strategic decisions by shippers to have export goods available ahead of a container becoming available. We aim to reveal the high potential of street turn strategies with SOCs when compared to the widely implemented immediate return policy. To this end, we develop a queueing model that can be used for empty container repositioning management by an importer – the consignee – to avoid unnecessary movements and hence improve overall utilization of scarce (empty) containers in the hinterland. Specifically, we model the system as a double-ended queue with a non-zero matching time and finite amount of resources to identify and carry out matches. The objective for the consignee is to determine how many containers should be stored to minimize its operational costs, including holding and travel costs. The exporter – the shipper – stores its goods with the expectation of a future match with an empty container from the consignee; when empty containers are scarce, it might make sense to capture any available empty container for export use. Thanks to advanced communication technologies, the shipper shares real-time information on the state of its available inventory for export. This allows the consignee to adjust its container withholding policy to the state of the shipper's available inventory.

We formulate the problem as a Markov decision process and prove in the single-truck case that the optimal inventory policy at the consignee is a state-dependent threshold policy with the optimal withholding threshold increasing with the inventory level at the shipper. This result is a novel contribution to the admission control literature, as doubled-ended queues with non-zero matching time have not been analyzed for this problem.

2 LITERATURE REVIEW

Empty container management received substantial attention from the transport and maritime economics communities. However, the transport and maritime economics literature does not take the inventory perspective into account, so their research paradigms cannot be used to tackle the problem considered in this paper. We refer to Dejax and Crainic (1987) for a review of early research from the operations management and transportation science communities on containerization and to Braekers et al. (2011), Lee and Meng (2014), and Lee and Song (2017) for recent surveys. Lee and Song (2017) divided existing contributions into two categories. The first investigates network flow models for the empty container repositioning issue (Li et al., 2007; Song and Dong, 2008; Dang et al., 2012, 2013; Hjortnaes et al., 2017). The second, more closely related to ours, considers the empty container repositioning problem from an inventory theory perspective. In these studies, the focus is generally on empty container movements between the sea terminal and consignee, but not with a street turn strategy involving an importer, exporter, and sea terminal as in this paper. We also note Li et al. (2004), Song and Zhang (2010), and Zhang et al. (2014), who considered a single empty depot located in a port and controlled by a shipping line, while Song (2005, 2007), Lam et al. (2007), Shi and Xu (2011), Ng et al. (2012), and Xie et al. (2017) focused on empty container (or equivalent vehicle) management for a two-depot system. As in this paper, Lam et al. (2007) and Shi and Xu (2011) employed Markov decision process approaches to determine optimal policies for inventory management but used different model assumptions than ours.

We model the consignee–shipper interaction as a double-ended queue in which the arrivals of customers and servers are independent processes. This definition enables considering the double-ended queue model for a wide range of applications such as shared-mobility systems (Xu et al., 2020; He et al., 2021), disaster and repair management (Di Crescenzo et al., 2012), passenger and taxi queues (Shi and Lian, 2016) and the allocation of live organs (Elalouf et al., 2018). Some studies focus on customer joining behavior (Shi and Lian, 2016; Wang et al., 2017; Jiang et al., 2021), performance evaluation (Conolly et al., 2002; Afèche et al., 2014; Diamant and Baron, 2019), and congestion control policies (Gurvich and Ward, 2015; Liu and Weerasinghe, 2021). However, matching times are assumed to be zero in the aforementioned papers; indeed, to the best of our knowledge, most previous studies do not consider matching time, as we do in this paper. We do note that Kim et al. (2010) developed a simulation procedure to derive performance measures. Also using a non-zero matching time, Shi et al. (2015) employed a matrix-analytic method to determine stability conditions and derive steady state probabilities. We focus instead on optimization issues and provide performance measures in closed form in certain particular cases. Next, Wang et al. (2023) analyzed a particular double-ended queue with a two-mass point distribution for the matching time. Finally, Nguyen and Phung-Duc (2022) studied customers' strategic joining decisions in a double-ended queue with a non-zero matching time for a passenger-taxi system, and extended their model to a situation with two types of customers in Nguyen and Phung-Duc (2023).

3 FORMULATION OF THE PROBLEM

We analyze the management of SOCs by a consignee in the hinterland. A consignee imports products via containers from a sea terminal. Later, empty containers are sent back to the sea terminal to be reloaded for a new import operation. To reduce road transportation costs, consignees can rent their containers to shippers in need of sending their products to the sea terminal. The policy of reusing containers for the return trip to the sea terminal is referred to as a street turn strategy, as opposed to an immediate return policy, in which all containers are sent back empty to the sea terminal. For simplicity of modeling, we assume that we have a single shipper.

Due to rental fees, a street turn strategy may reduce transportation costs. Assume that the transportation cost on roads is t monetary units per kilometer, which is either paid to a transport company or through direct costs if the trucks are owned by the consignee, and that the shipper may be charged a renting fee of r per time unit of container use. If the distance between the sea terminal and the consignee's location is d_1, the distance (out and back) that must be covered to deliver a container from the consignee to the shipper is d_2, and the distance between the shipper and the sea terminal is d_3, then a necessary condition on average costs to consider a street turn strategy is

$$td_1 > td_2 - rd_3/v, \tag{1}$$

where v is the average driving speed in kilometers per time unit. It should be noted that the rental fee r is only applied on d_3, indicating that the consignee is in charge of transporting the empty container to the shipper's location and that the shipper only pays for the time it uses the container. Condition 1 is however only a necessary condition to implement a street turn strategy that is valid for a given container when reuse is possible. Considering the imbalance between imports and exports, a full reuse of containers may not be feasible, so it may be necessary to determine which containers should be immediately returned to the sea terminal and which should be kept at the consignee's location for future reuse. The function that associates the decision to store or return a container with each state of the system is called a withholding policy.

For simplicity, we assume that the containers' arrival process at the consignee is Poisson with constant parameter λ_c. The Poisson assumption is justified for the arrival processes at sea terminals. Some statistical analyses have revealed that vessel arrivals fit well with a Poisson distribution (Plumlee, 1966; Kozan, 1997). In addition, truck arrivals at the sea terminal can be modeled by Poisson distributions (Minh and Van Noi, 2021; Roy et al., 2022). We make the further assumption that the arrival rate is constant over time. This may not be realistic because in some areas there is a pronounced variation by time of day and day of the week, due to the activity at the sea terminal. If time dependency varies slowly relative to the system dynamics, then such systems have been typically analyzed using a pointwise stationary approximation, where the performance at a given time is approximated by the steady-state performance of the stationary system with a constant arrival rate. The latter is given by the mean arrival rate in a given interval around the observation point (Green and Kolesar, 1991; Jennings et al., 1996).

The non-focal shipper needs to send its loads to the shipping line. It either asks the shipping line to send an empty container or rents an empty container from the consignee. The need for empty containers at the shipper follows a Poisson process with rate λ_s. We assume that it is cheaper for the shipper to rent a container from the consignee than to request a container from the shipping line and pay the resulting high detention fees. The time to send a container from the consignee's location to the shipper is non-zero because it includes the time for the consignee and shipper to make an agreement, the time to find an available truck, and the transportation time between the consignee and shipper. To account for the variability of these durations, we assume that the total time to send a container from the consignee's location to the shipper, known as the matching time, is exponentially distributed with rate μ. The high variability of the exponential distribution is also one way to model the diversity of shippers' locations, allowing us to consider the model in the present study as an approximation of the multi-shipper problem. We further assume that there are m trucks devoted to the matches, which creates a bound on the number of simultaneous matches that can be carried out.

Furthermore, the shipper stores part of its stockpile for future matches. Specifically, the shipper maintains an inventory level equivalent to q container volumes. If $q = 0$, the shipper declines to reuse containers from the consignee. If $q > 0$, then a quantity of q equivalent containers is stored at the shipper's location, either through matching processes or waiting for a match to occur. This policy is simple to implement by the shipper and allows a control of the time spent by the goods in the hinterland.

The objective for the consignee is to determine the optimal withholding policy that minimizes the operational cost per time unit of a street turn strategy, denoted by C. We denote by λ_r the expected rate of containers that are immediately returned to the sea terminal; therefore, the average total travel cost of the street turn strategy T is $T := \lambda_c t d_1 + (\lambda_c - \lambda_r)(td_2 - rd_3/v)$. In addition to the travel cost, a street turn policy induces the storage of empty containers at the consignee's location, which results in an expected holding cost H, expressed as $H := hN_I$, where h is the holding cost per time unit and per container, and N_I is the expected number of containers at the consignee's location. The operational cost per time unit is then defined as

$$C := \lambda_r t d_1 + (\lambda_c - \lambda_r)(td_2 - rd_3/v) + hN_I. \tag{2}$$

Note that with a sufficiently high rental fee, the cost can become negative, indicating that the street turn policy can generate revenue for the consignee.

We are also interested in the consequences of the street turn strategy on the use of containers, as measured by the matching proportion and occupation rate of containers. The matching proportion, P_m, is the ratio between the number of matches per time unit divided by λ_s; that is, $P_m := \frac{\lambda_c - \lambda_r}{\lambda_s}$. This means that the matching proportion represents the proportion of the goods sent by the shipper to the sea terminal using a container from the consignee. The occupation rate of containers, also called utilization rate and designated as U, is measured by the proportion of time during which a container delivered at the consignee is full, assuming that containers sent back empty to the sea terminal are full $\frac{d_1}{2d_1} = 50\%$ of the time, while containers that are used for matches are full $\frac{d_1+d_3}{d_1+d_2+d_3}$ of the time. Therefore, we define the occupation rate as $U := \frac{0.5\lambda_r + (\lambda_c - \lambda_r)\frac{d_1+d_3}{d_1+d_2+d_3}}{\lambda_c}$. A summary of the notations is given in Figure 1.

4 WITHHOLDING POLICY

We determine the optimal withholding policy. The consignee is informed by the shipper about the volume of goods waiting for a match. Based on this information, the consignee decides to either immediately return an arriving container to the sea terminal or to store it for a future match. The function, which associates the decision to store or return with an arriving container at a given state of the system, is called a dynamic withholding policy. We employ a Markov decision process approach to prove that the optimal policy is a state-dependent threshold policy, where the admission threshold depends on the system state at the shipper.

We define the system state by (x, y), where x is the number of containers present in the system, either at the consignee's location or carrying out a match, and y is the volume of production stored by the shipper in equivalent container volume (either carrying out a match or waiting for a future match), with $x \in \mathbb{N}_0$ and $y = 0, 1, \cdots, q$. Note that we do not need to distinguish between containers that are waiting at the consignee's location and containers that are carrying out a match. Since we cannot simultaneously have a container waiting at the consignee, a container waiting for a match at the shipper, and an available truck, we deduce that the number of trucks traveling from the consignee to the shipper is $\min(x, y, m)$, so the number of containers that are waiting at the consignee's location is $x - \min(x, y, m)$, and the number of equivalent container volume that are waiting for a future match at the shipper is $y - \min(x, y, m)$.

The transition rate from state (x, y) to state (x', y') is denoted by $r_{(x,y),(x',y')}$ for $x \in \mathbb{N}_0$ and $y = 0, 1, \cdots, q$. It is λ_c if $x \in \mathbb{N}_0$ and $y = 0, 1, \cdots, q$, with $(x', y') = (x+1, y)$ if action store is selected. It is λ_c if $x \in \mathbb{N}_0$ and $y = 0, 1, \cdots, q$, with $(x', y') = (x, y)$, if action return is selected. It is λ_s if $x \in \mathbb{N}_0$ and $y = 0, 1, \cdots, q-1$, with $(x', y') = (x, y+1)$. It is $\min(x, y, m)\mu$ if $x \in \mathbb{N}_1$ and $y = 1, 2, \cdots, q$, with $(x', y') = (x-1, y-1)$. Finally, it is 0 otherwise.

If return is selected, a transportation cost of td_1 is incurred, while if store is selected, the transportation cost is $td_2 - rd_3/v$. The number of containers that are waiting for a future match at the consignee's location is $x - \min(x, y, m)$, so the holding cost in state (x, y) is $h(x - \min(x, y, m))$.

The total event rate $\lambda_s + \lambda_c + m\mu$ is bounded. Therefore, we use the uniformization technique (Puterman, 1994) because it enables us to consider the continuous time Markov chain as a discrete time one, assuming that $\lambda_s + \lambda_c + m\mu = 1$. By adding a fictitious transition from a state to itself, we define the system's value function, $V_k(x, y)$, over k steps by $V_0(x, y) = 0$, and for $k \geq 0$

$$\begin{aligned} V_{k+1}(x,y) &= h(x - \min(x,y,m)) + \lambda_c \min(V_k(x+1,y) \\ &+ td_2 - rd_3/v, V_k(x,y) + td_1) + \lambda_{sy<q} V_k(x,y+1) \\ &+ \min(x,y,m)\mu V_k(x-1,y-1) \\ &+ (1 - (\lambda_c + \lambda_{sy<q} + \min(x,y,m)\mu)) V_k(x,y), \end{aligned} \tag{3}$$

with $x \in \mathbb{N}_0$, and $y = 0, 1, \cdots, q$.

The minimizing operator at the first line of (3) represents the control action to either store or return a container. We obtain the long-run average optimal actions by applying the value iteration technique through the recursive evaluation of V_k, using (3) for $k \geq 0$. As k tends to infinity, the optimal policy converges to the unique average optimal policy that minimizes the consignee's operational cost, and the difference $V_{k+1}(x, y) - V_k(x, y)$ converges to the long-run optimal cost (Puterman, 1994).

In Theorem 1, we prove the threshold form of the optimal policy for $m = 1$. We observe numerically that the

System state	
x	Number of containers at the consignee's location or carrying out a match
y	Amount of goods at the shipper in container volumes
System parameters	
λ_c	Containers' arrival rate at the consignee's location
λ_s	Containers' demand rate at the shipper's location
μ	Matching rate
m	Number of trucks to carry out matches
c, s	Ratios λ_c/μ and λ_s/μ
b	Ratio λ_c/λ_s
q	Maximum inventory for the shipper to hold (expressed in container equivalents)
n	Fixed admission withholding threshold for empty containers (i.e., maximum number of containers for the consignee to keep in the hinterland)
n_y	State-dependent admission withholding thresholds for empty containers (i.e., maximum number of containers for the consignee to keep in the hinterland when y equivalent container volumes are at the shipper)
Cost parameters and distances	
t	Transportation cost on roads in monetary units per kilometer
h	Holding cost per time unit and per container
r	Rental fee per time unit of container use
v	Average driving speed in kilometers per time unit
d_1	Distance between the sea terminal and the consignee's location
d_2	Distance that must be covered to deliver a container from the consignee to the shipper
d_3	Distance between the shipper and the sea terminal
Performance measures	
$p_{x,y}$	Stationary probability to be in state (x, y)
λ_r	Rate of empty containers returned immediately to the sea terminal per time unit
N_l	Expected number of containers at the consignee's location
H	Expected holding cost per time unit
T	Expected travel cost per time unit
C	Expected operational cost per time unit ($C = H + T$)
P_m	Matching proportion
U	Utilization rate of containers
$V_k(x, y)$	Value function over k steps in state (x, y)

Figure 1: **Notations**

same result holds when $m > 1$. The idea is to prove that the threshold structure of the optimal policy is kept in the induction step from k to $k+1$. In other words, we prove that if it is optimal to return a container in state (x,y), then the same action is optimal in state $(x+1,y)$. At iteration k, a necessary condition for this is that if $V_k(x+1,y)+td_2-rd_3/v-V_k(x,y)-td_1 \geq 0$, then $V_k(x+2,y)+td_2-rd_3/v-V_k(x+1,y)-td_1 \geq 0$, or equivalently $V_k(x+2,y)+td_2-rd_3/v-V_k(x+1,y)-td_1 \geq V_k(x+1,y)+td_2-rd_3/v-V_k(x,y)-td_1$, which can be rewritten as

$$V_k(x+2,y)+V_k(x,y)-2V_k(x+1,y) \geq 0.$$

Therefore, by showing that $V_k(x,y)$ is convex in x for each k, we prove that the optimal policy converges to the unique average optimal policy, defined by the withholding thresholds n_y for $y = 0,1,\cdots,q$, such that an empty container is admitted in the system in state (x,y) if and only if $x < n_y$. However, the convexity property in x of $V_k(x,y)$ cannot be proven in isolation but has to be proven with a set of other properties. This set of properties $\mathcal{C}$ is defined for a given function f as follows:

$$\text{Increasing in } x: f(x+1,y) \geq f(x,y), \tag{4}$$
$$\text{Decreasing in } y: f(x,y+1) \leq f(x,y), \tag{5}$$
$$\text{Increasing in } (x,y): f(x+1,y+1) \geq f(x,y), \tag{6}$$
$$\text{Convex in } x: f(x+2,y)+f(x,y) \geq 2f(x+1,y), \tag{7}$$
$$\text{Convex in } y: f(x,y+2)+f(x,y) \geq 2f(x,y+1), \tag{8}$$
$$\text{Submodular in } (x,y): \tag{9}$$
$$f(x,y+1)+f(x+1,y) \geq f(x+1,y+1)+f(x,y), \tag{10}$$
$$f(x+2,y+1)+f(x,y)-f(x+1,y) -f(x+1,y+1) \geq 0, \tag{11}$$
$$f(x+1,y+2)+f(x,y)-f(x,y+1) -f(x+1,y+1) \geq 0. \tag{12}$$

From the submodularity property of $V_k(x,y)$, we deduce that the thresholds n_y are increasing in y. Formally, having n_y increasing in y means that if an empty container is accepted in the system in state (x,y), then it should also be accepted in the system in state $(x,y+1)$. This means that if $V_k(x+1,y)+td_2-rd_3/v \leq V_k(x,y)+td_1$, then $V_k(x+1,y+1)+td_2-rd_3/v \leq V_k(x,y+1)+td_1$. This implication holds if $V_k(x,y)-V_k(x+1,y)+td_1-(td_2-rd_3/v) \leq V_k(x,y+1)-V_k(x+1,y+1)+td_1-(td_2-rd_3/v)$. This inequality is equivalent to the submodular property of $V_k(x,y)$.

This result has an intuitive explanation. The state variable y is the amount of goods in equivalent container units at the shipper. If $x \geq y$, then $x-\min(y,m)$ is the number of stored containers that are waiting for a match at the consignee's location. The holding cost associated with $x-y$ is decreasing in y, which tends to incentivize the decision to admit a container when y is large. When $x < y$, then y may also include the goods that are waiting for a future match at the shipper. Since a system with a larger inventory at the shipper has more capacity to carry out matches, a large y is again an incentive for the consignee to store containers.

Theorem 1 (Optimal withholding policy)**.** *In the single-truck case (i.e., $m = 1$), the optimal dynamic withholding policy for the consignee is a state-dependent threshold policy with parameters n_y, for $y = 0, 1, \cdots, q$, which are increasing in y, such that an arriving container is admitted into the system in state (x, y) if and only if $x < n_y$. If $x \geq n_y$, an arriving container is returned directly to the sea terminal.*

Theorem 1 can be proven by induction.For the sake of brevity, we skip the proof. Of particular note in the result of Theorem 1 is that the consignee exerts control over the number of containers in the inventory *and* the number of containers carrying out a match, not only on the number of containers in inventory, as one might expect.

5 Numerical Experiments

We study in this section the effect of the traffic intensity on the optimal policy and the system performance. The numerical results are shown in Figure 2 for $\lambda_c + \lambda_s = 40, 80, 160$, and 320 container equivalents per day in export-dominant, balanced and import-dominant areas.

Traffic intensity influences the withholding policy and cost savings almost as much as an imbalance between exports and imports. Specifically, the thresholds decrease with traffic intensity. Storing containers is one way to avoid a shortage when there is a demand from a shipper. When the intensity of container arrivals is high, the time until a demand can be fulfilled is brief even if there is an apparent shortage, so the need to store containers to reduce the impact of an immediate shortage is low, which explains why thresholds decrease with the intensity of demand. Due to a large number of containers entering the hinterland, costs increase with the intensity of demand, which also shows that the absolute savings are greatest in high-traffic areas. However, this is only true in absolute terms. The relative difference between the immediate return and optimal withholding policies is the greatest in low-traffic areas, where storing containers has the potential to compensate for long intervals between arrivals. Moreover, the proportion of immediately returned containers and the proportion of matches are both heavily impacted by the intensity of $\lambda_c + \lambda_s$. With high traffic, even in export-dominant areas, the proportion of returned containers is high. This is due to the decision to set very low withholding thresholds, such that most matches cannot be carried out. Increasing withholding thresholds would increase holding costs without influencing the volume of immediately returned containers (which is constrained here by the shipper's low inventory level).

6 Conclusions

We analyzed the management of SOCs by a consignee in the hinterland with the aim of minimizing the sum of travel and holding costs. To this end, we formulated the problem as a Markov decision process for a double-ended queue with a non-zero matching time and finite amount of resources, where the consignee decides to either store a container for future reuse by an exporter or return it immediately to the sea terminal. We proved in the single-truck case that the optimal withholding policy is a state-dependent threshold policy with threshold levels increasing along with shipper inventory. We showed that traffic intensity plays almost the same role as the balance between imports and exports, revealing that the role of inventory at the consignee's location is to compensate for the scarcity of containers in the hinterland.

As future research, the assumptions made can be modified by considering non-Poisson processes for the arrival of and demand for containers or a non-exponential distribution of matching times. Although changing distributions may lead to different quantitative results, it is unlikely to modify the insights provided by the Markovian analysis. We could also consider a situation with multiple consignees and shippers.

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λ_c	λ_s	C^*	n_0	n_1	$\frac{C^*-\lambda_c t d_1}{\lambda_c t d_1}$	$\frac{H}{C^*}$	$\frac{\lambda_r}{\lambda_c}$	P_m	U
12.5	27.5	€2,475.51	16	16	-44.056%	1.986%	9.899%	40.955%	77.030%
20	20	€5,346.07	5	5	-24.491%	0.348%	50.621%	49.379%	64.814%
27.5	12.5	€8,393.65	3	3	-13.779%	0.136%	72.280%	60.984%	58.316%
25	55	€6,315.62	7	7	-28.637%	0.408%	42.294%	26.230%	67.312%
40	40	€11,840.33	4	4	-16.382%	0.123%	67.116%	32.884%	59.865%
55	25	€17,530.16	2	3	-9.963%	0.051%	80.034%	43.925%	55.990%
50	110	€14,763.91	5	5	-16.588%	0.125%	66.701%	15.136%	59.990%
80	80	€25,537.95	3	3	-9.824%	0.039%	80.334%	19.666%	55.900%
110	50	€36,448.01	2	3	-6.400%	0.026%	87.186%	28.190%	53.844%
100	220	€32,215.90	3	4	-8.995%	0.043%	81.979%	8.192%	55.406%
160	160	€53,546.71	2	3	-5.461%	0.018%	89.073%	10.927%	53.278%
220	100	€74,974.44	2	2	-3.731%	0.007%	92.544%	16.404%	52.237%

Figure 2: **Impact demand intensity** ($m = 1$ **truck,** $q = 1$ **equivalent container,** $d_1 = d_3 = 200$ **km,** $d_2 = 100$ **km,** $v = 90$ **km/hour,** $\mu = \frac{v}{1.1 d_2}$**,** $h = r =$ **€5/day, and** $t =$ **€1.77/km)**

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