Integrated Maintenance-Quality policy considering variable production cost

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***Abstract* – This paper deals with an integrated maintenance-quality strategy in response to production costs that rise as the production system state degrades and affects the quality of the output products. We consider a single machine with an increasing failure rate producing high quality items termed first-rate products, substandard quality items termed second-rate products, and non-conforming items. Rework activities are possible for second-rate and non-conforming products to improve their quality and sell them at an appropriate price. A preventive maintenance (PM) strategy with minimal repair at failure is proposed. It involves performing imperfect PM actions after a certain number of produced batches and then, as the cost and duration of these actions increase with time, a perfect maintenance action (overhaul) is undertaken after a certain number of imperfect PM actions. An analytical model is developed in order to determine both: the optimal number of batches produced before scheduling each imperfect PM, and the number of imperfect PM actions to be undertaken before performing an overhaul. These optimal values maximize the total expected profit, taking into account revenues as well as costs related to production, maintenance and reworking. Numerical examples are presented in order to illustrate the use of the developed model for deriving the optimal integrated strategy for any instance of the problem.**

***Keywords* – Maintenance strategies, imperfect preventive maintenance, reworking, quality, degradation, variable production cost.**

1 INTRODUCTION AND LITERATURE REVIEW

Manufacturing companies are facing serious challenges in today's competitive world. Since there are ever more obstacles to business success, organizations are increasingly being forced to improve the quality of their internal processes and systems. To achieve this, there is renewed emphasis on strategies that consider a global systemic approach reflecting the interactions between several functions of the company (maintenance, quality, marketing, production, inventory, delivery etc.). In this context, several researchers have been interested in developing policies which treat different functions concurrently, such as maintenance/production, maintenance/quality, production/delivery etc. After decades during which these aspects had been studied separately.

Looking to the research related to reliability and maintenance, we note that maintenance policies were first introduced in the literature by Barlow and Hunter (1960). They have since been studied by many other researchers, as can be seen in a survey on maintenance models by Wang (2002). Simeu-Abazi and Sassine (2001) proposed an effective way of modeling complex manufacturing systems through hierarchical and modular analysis using stochastic Petri nets and Markov chains. This was to allow various maintenance strategies to be coded in the generic model with the aim of studying their influence on system dependability and performance. In the same context, Savsar (2004) analyzed the effects of corrective, preventive, and opportunistic maintenance policies on the performance of a Flexible Manufacturing System by using an approach that combines simulation and analytical models.

Maintenance actions can be classified according to three types, reflecting the degree to which the operating conditions of a piece of equipment is restored after a maintenance activity: perfect, minimal and imperfect maintenance. With perfect maintenance, the system is restored to the 'as good as new' state, which means that the system now has the same lifetime probability distribution and failure rate function as a new identical equipment. With minimal maintenance, the system is restored to the failure rate it had just before failure; the operating system state becomes, to use the expression largely adopted in the literature, 'as bad as old'. The third type of maintenance actions, imperfect maintenance, restores the system to an operating state between the two previous ones ('as good as new' and 'as bad as old'). Many researchers consider this type of preventive maintenance as the most realistic. Various methods and mathematical models have been proposed to estimate the reliability measures and to determine the optimum maintenance policies for imperfect maintenance of single and multi-component systems, as can be seen in an extensive review Pham and Wang (1996) on imperfect maintenance policies published between 1985 and 1996. During the last two decades, researchers and managers recognized that strategies dissociating complementary functions (particularly production, maintenance and quality)

were ineffective. Attention has been brought to developing integrated multiple functions strategies. In fact, many integrated models have been pro-
posed in the literature to examine various interactions between different pairs of functions including maintenance, production, lot sizing and quality (Ben-Daya (1999), Cassady et al. (2000), Chelbi and Ait-Kadi (2004), Linderman et al. (2005), Dellagi et al. (2007)).

Ben-Daya (2002) proposed an integrated model emphasizing economic production lot sizing and imperfect maintenance for an imperfect process having a general deterioration distribution with an increasing hazard rate.

Radhoui et al. (2010) dealt with the problem of integrating preventive maintenance, production, and quality control policies for a manufacturing system producing conforming and non-conforming items. They determined, simultaneously, the optimal rejection rates thresholds level and the buffer stock size while minimizing the average total cost per time unit.

Budai et al. (2008) presented an overview of mathematical models that consider interactions between production and maintenance. They described industry sectors in which these interactions have been studied.

Dinis et al. (2019) presented a framework for the qualitative and quantitative characterization of maintenance activities to guide Maintenance, Repair and Overhaul (MRO) organization to perform capacity planning and scheduling. The quantitative study is based on maintenance projects from a Portuguese aircraft company. It proved the impact of the stochastic part of maintenance on the planning activity. To manage this uncertainty, they used methods for data treatment and data analysis. They developed a method entitled Dimensional Maintenance Data Analysis (3D-MDA). It is based on a space-time-skill coordinate system in which indicators are calculated from historical data to comprehensively characterize the expected maintenance work.

Brandolese et al. (1996) proposed an expert system for the planning and management of a multi-product and one-stage production system made up of flexible machines operating in parallel. Their system schedules both production and maintenance at the same time in a way that the preventive maintenance tasks are integrated into the planning and placed as close as possible to the optimal maintenance periods.

Nourelfath et al. (2010) examined a case of joint preventive maintenance and production planning, for a production system composed of a set of parallel components. In Nourelfath and Châtelet (2012), the authors assumed the presence of economic dependence and common cause failures in the production system.

Mifdal et al. (2015) considered the same problem in the case of multiple-products manufacturing systems. Recently, Shamsaei and Van Vyve (2017) formulated integrated maintenance scheduling and production planning problems as mixed-integer linear programs. They demonstrated the efficiency of the proposed approach to solve problems of up to 10 products and 24 periods, sizes that were simply unreachable before.

Salari and Makis (2017) studied a system composed of N identical and independent units subject to progressive deterioration. They proposed two condition-based maintenance policies and developed models integrating maintenance, production and demand. They determine the optimal parameters for each strategy minimizing average cost per time unit. The proposed problem is formulated and solved in the semi-Markov decision process (SMDP) framework.

Chelbi et al. (2008) proposed a joint production-maintenance strategy for unreliable production systems producing conforming and non-conforming items; an age-based PM policy is proposed. They determine both the production lot-size and the age for preventive maintenance. They showed that performing preventive maintenance leads to a reduction in inventory and quality-related costs.

Tambe and Kulkarni (2015) used a genetic algorithm in order to optimize decisions around maintenance and quality control, given the machine's production schedule.

Nourelfath et al. (2016) put forward a joint production, maintenance, and quality strategy for an imperfect process in the context of a multi-period/ multi-product capacitated lot sizing in order to minimize the total cost (maintenance, quality, and production) while satisfying long-term product demand. PM actions were assumed to be imperfect, reducing the age of the system proportionally to the PM level. Various other works dealt with the simultaneous integration of maintenance, quality, and production as presented in a review of the literature by Ben-Daya and Rahim (2001).

In this paper, we consider specific situations of production systems whose degradation affects the quality of the output products and increases production costs. As a response to that, we propose an integrated maintenance-quality strategy.

The remainder of the paper is organized as follows: in Section 2, we detail the proposed strategy. In section 3, we present an analytical study in which we provide the notations used and the proposed mathematical model. In section 4, a numerical study is presented in order to illustrate the use of the proposed model. Finally, a conclusion and some perspectives are presented in section 5.

2 PROBLEM STATEMENT

We consider a repairable manufacturing system composed of a single machine subject to an increasing random failure rate following a Weibull distribution with a shape parameter $\beta > 1$ and a scale parameter $\varepsilon > 0$. Such random deterioration impacts not only the production system but also the product quality. We distinguish, then, three different types of output products. First-rate items are considered items of the highest quality; a batch is to be sold at the best price P_{max}. Second-rate products are items of sub-standard quality, but their quality problems do not affect their main function. They can be reworked to be sold at the best price. Finally, there are non-conforming items (figure 1). These products, similarly, will be reworked in order to ameliorate their quality and to sell them at an appropriate price. They can be restored to the best quality condition or to the second-rate condition and sold or they can be rejected. Reworked batches thus cannot always be sold at the best price P_{max}. Consequently, we consider that overall, batches are sold at an average price P'_{max} which represents a fraction of the best price Pmax ($P'_{max} = \tau \times P_{max}$) where
τ is a strictly decreasing positive function depending on the number of batches produced and reworked. In fact, τ

represents a 'depreciation factor'. It is there in order to take into consideration products that cannot be restored to the best quality condition and sold at the best price. It is evident that, as the production system state degrades, the increase in the number of produced batches results in the rise of the non-conformity rate of output products which translates into a decrease in the Safety factor according to the number of produced batches. As such, the depreciation factor and the selling price of non-conforming items, can be expressed as follows:

$$\tau = e^{-\rho \times N} \quad (1)$$

$$P'_{max} = \tau \times P_{max} = e^{-\rho \times N} \times P_{max} \quad (2)$$

With N representing the number of batches produced and ρ being a strictly positive constant that can be estimated empirically.

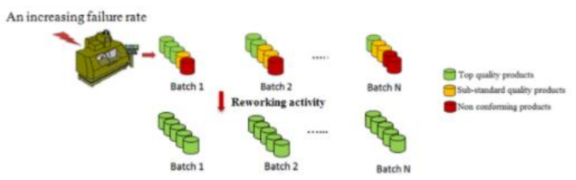

Figure 1. The proposed preventive maintenance strategy

As stated above, to improve the quality condition of second-rate products and non-conforming items and sell them at an appropriate price, reworking activities are proposed. There is interdependency between the manufacturing system's condition and the quality of output products, and so the reworking task and its associated cost are related to the progressive degradation of the manufacturing system Gouiaa-Mtibaa et al. (2017). In order to reduce the probability of failures and their impact on the output product quality, an imperfect preventive maintenance policy, with minimal repair at failure, is proposed. It consists in performing an imperfect PM action after producing N batches. After (h − 1) imperfect PM actions, an overhaul or renewal maintenance action is undertaken in order to restore the system to 'as good as new' state.

The following assumptions are stated:

- The planning horizon is infinite.
- A single type of product is considered.
- The quality degradation rate is random. It depends on the system's failure rate.

3 THE ANALYTICAL MODEL

3.1 *Notations*

The following notations will be used throughout the paper:

T: The average duration of a renewal cycle
T': The average duration of a PM sub-cycle
N: The number of batches produced before performing an imperfect PM action (decision variable)
h: The number of PM sub-cycles at the end of which an overhaul is performed (decision variable)
PT [N, h]: The average total profit per time unit
PT [N]: The average total profit per time unit when the other decision variable h is fixed
PT [h]: The average total profit per time unit when the other decision variable N is fixed
R: The total revenue over a cycle
P_{max}: The selling price of a batch with the best quality (100% conforming items)
P'_{max}: The mean selling price of a reworked batch
M_c: The average cost of minimal repair
C_{cm}: The total expected cost of repair over a cycle
Nb_{fk}: The mean number of failures during a PM sub-cycle k
$M'_{p,k}$: The cost of the k^{th} imperfect preventive maintenance action
C_{pm}: The total preventive maintenance cost over a cycle
C_p: The total production cost
$C_{p,i}$: The production cost of the i^{th} batch
C_{pu}: The unitary production cost
C_{rk}: The total reworking cost over a cycle
$C_{rk,i}$: The reworking cost of the i^{th} batch
C_{rku}: The average unitary reworking cost
C^k_{rw}: The cost of a renewal if it is performed after k imperfect preventive maintenance activities
C_{rw}: The renewal cost
μ_p: The duration of the first imperfect preventive maintenance action
$\mu_{p,k}$: The duration of the k^{th} imperfect preventive maintenance action
μ'_p: The duration of an overhaul (renewal)
μ_c: The average duration of a minimal repair
Proc: The average processing time to produce one batch
F(t): The probability distribution function associated with the time to failure of the production system (supposed to be a Weibull distribution)
g(t): The probability density function associated with the length of repair time (supposed to be an Exponential distribution)
λ(t): The failure rate function
β : The Weibull shape parameter
ε : The Weibull scale parameter
η : The Exponential function parameter
τ : The depreciation factor
ψi: The reworking ratio of the i^{th} batch
θ : The renewal factor
N_{RC} : The number of batches produced during a renewal cycle

3.2 *Mathematical development*

The expected renewal cycle length T includes the production times, the imperfect preventive maintenance actions duration, the total average minimal repair time and the renewal duration. The cycle T contains h PM sub-cycles
(T'). A PM sub-cycle includes the production time of N batches and the average repair time. At the end of the first h PM sub-cycles, a renewal is performed. Figure (2) below shows an example of a renewal cycle with 4 PM sub-cycles and 3 imperfect preventive maintenance actions.

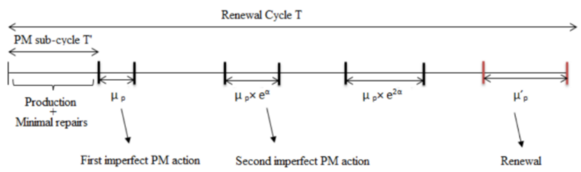

Figure 2. Representation of a renewal cycle

Due to the progressive degradation of the manufacturing system and to the imperfection of PM, we consider that the time to perform PM increases from one action to the next one.

The duration of an imperfect PM action is equal to the duration of the previous one multiplied by 'The evolution factor of the preventive maintenance tasks' $e^{\alpha 1}$, α_1 is a strictly positive parameter that can be estimated empirically. Consequently, the relation between the duration of consecutive imperfect preventive maintenance actions is expressed as follows:

$$\mu_{p,k} = \mu_{p,k-1} \times e^{\alpha_1} \qquad \forall k \in 1..(h-1) \tag{3}$$

At the end of the last PM sub-cycle, a renewal activity is performed taking a time μ_p. The expected duration of a cycle is therefore given by the following equation (4).

$$T = \sum_{k=1}^{h} N \times \text{Proc} + \mu_c \times Nb_{fk} + \sum_{k=2}^{h} \mu_p \times e^{(k-2)\times\alpha_1} + \mu'_p \tag{4}$$

The expected total profit per time unit PT [N, h] is calculated by considering the difference between the total revenue and the sum of the costs related to maintenance, production, reworking and renewal, divided by the expected cycle length T. In addition, we assume that on one hand, the sum of the reworking cost of a batch i and its production cost must not exceed the selling price of a batch, and on the other hand, the preventive maintenance cost does not exceed the renewal cost. These conditions are expressed by constraints (6) and (7) below.

$$PT[N,h] = \frac{R - (C_{pm} + C_{cm} + C_p + C_{rk} + C_{rw})}{T} \tag{5}$$

With

$$C_{rk,i} + C_{p,i} \le P'_{\max} \; \forall i \in 1..N \tag{6}$$

and

$$M'_{p,k} \le C^k_{rw} \; \forall k \in 1..h \tag{7}$$

We develop below the expressions of the different terms presented in equation (5).

3.2.1 Total revenue

During a cycle T, we have h PM sub-cycles and over every PM selling price P_{max}. Consequently, the total revenue is given by :

$$R = \sum_{k=1}^{h} N \times P'_{\max} = \sum_{k=1}^{h} N \times e^{-\rho \times N} \times P_{\max} \tag{8}$$

3.2.2 Preventive maintenance cost over a cycle

During a cycle, (h-1) imperfect preventive maintenance actions are performed and followed by a single renewal activity. Considering the increasing duration of PM actions with time (equation 3), the relation between the cost of consecutive preventive actions is:

$$M'_{p,k} = M'_{p,k-1} \times e^{\alpha_1} \; \forall k \in 1..(h-1) \tag{9}$$

Thus, the total preventive maintenance cost (C_{pm}) is obtained by determining the sum of the costs of (h-1) imperfect preventive maintenance actions:

$$C_{pm} = \sum_{k=2}^{h} M'_p \times e^{(k-2)\times\alpha_1} \tag{10}$$

3.2.3 Average repair cost over a cycle

The equipment is subject to minimal repair following failures between PM ac tions during a given cycle made of h PM sub-cycles. Hence, the expected cost of minimal repairs over a cycle is given by:

$$C_{cm} = \sum_{k=1}^{h} M_c \times Nb_{fk} \tag{11}$$

Nb_{fk} being the mean number of failures over of a PM sub-cycle k. The relation between the average number of repairs over consecutive PM sub- cycles (k and k + 1) is given by the following expression:

$$Nb_{fk+1} = e^{\alpha_2} \times Nb_{fk} \tag{12}$$

With $e^{\alpha 2}$ representing a degradation factor. It can be estimated empirically. Consequently, the mean number of failures Nb_{fk} during a PM sub-cycle k is given by Gouiaa – Mtibaa et al. (2017):

$$Nb_{fk} = Nb_1 \times e^{(k-1)\times\alpha_2} \tag{13}$$

We recall that repair times have non negligible durations. The mean number of failures (minimal repairs) is based on the convolution product of the time to failure distribution and the repair time distribution (Lim et al. (2005)). The average number of failures Nb_{f1} during the first PM sub-cycle [0,T'] is given by :

$$Nb_1 = -\log\left(1 - \int_0^{T'} F(t-x) \times g(x)\mathrm{d}x\right) \tag{14}$$

Consequently, the mean number of failures during a given PM sub-cycle k is given by:

$$Nb_{fk} = \left[-\log\left(1 - \int_0^{N\times Proc} F(N \times \text{Proc} - x) \times g(x)\mathrm{d}x\right)\right] \times e^{(k-1)\times\alpha_2} \tag{15}$$

We recall that F(t) is a Weibull probability distribution function associated with the time to failure with a shape parameter β and a scale parameter ε, and g(t)is an exponential probability density function associated with repair time with parameter η. Hence, the expected number of minimal repairs in a sub-period k is expressed as follows:

$$Nb_{fk} = -\log\left(1 - \int_0^{N\times Proc}\left(1 - e^{-\left(\frac{N\times Proc - x}{\epsilon}\right)^{\beta}}\right) \times \eta e^{-\eta x}\,\mathrm{d}x\right) \times e^{(k-1)\times\alpha_2} \tag{16}$$

Consequently, the expected cost of minimal repairs over a cycle is obtained by:

$$C_{cm} = \sum_{k=1}^{h} M_c \times \left[-\log \left(1 - \int_0^{N\times Proc} \left(1 - e^{-\left(\frac{N\times Proc - x}{\epsilon}\right)^{\beta}} \right) \times \eta e^{-\eta x} \, dx \right) \right] \times e^{(k-1)\times \alpha_2} \tag{17}$$

3.2.4 Total reworking cost over a cycle

Over a cycle T, we have h PM sub-cycles and over every PM sub-cycle, second-rate and non-conforming products have to be reworked in order to improve their quality condition. The reworking cost of a batch is obtained by multiplying the average unit reworking cost by the reworking ratio ψ_i :

$$C_{rk,i} = C_{rku} \times \psi_i \tag{18}$$

Where,

$$\psi_i = \frac{\lambda_i}{\lambda_{N+1}} \; \forall i \in 1..N \tag{19}$$

And λ_i represents the failure rate after producing the i^{th} batch. This expression of the reworking ratio is detailed in (Gouiaa-Mtibaa et al.(2017)). Thus, the total reworking cost is given by:

$$C_{rk} = \sum_{k=1}^{h} \left(\sum_{i=1}^{N} C_{rku} \times \left(\frac{\lambda_i}{\lambda_{N+1}} \right) \right) \tag{20}$$

3.2.5 The renewal cost

A renewal is performed at the end of a cycle. The later it is undertaken, the more it costs. We consider an evolution factor, θ, of this cost. Formally: $C_{rw}^{h} = C_{rw}^{h-1} \times e^{\theta}$

The renewal cost is therefore expressed by:

$$C_{rw} = C_{rwu} \times e^{(h-1)\times\theta} \tag{21}$$

3.2.6 Total production cost over a cycle

We consider an increasing production cost due to the progressive degradation of the production system (increase of energy consumption, excessive use of lubricants, etc.). We model the production cost of the i^{th} batch with the following expression considering the system's failure rate:

$$C_{p,i} = C_{pu} + C_{pu} \times e^{-\frac{\lambda_i}{\lambda_{i-1}}} = C_{pu} \times \left(1 + e^{\left(-\frac{\lambda_i}{\lambda_{i-1}}\right)} \right) \tag{22}$$

Consequently, the total production cost during a cycle is given by:

$$C_p = \sum_{k=1}^{h} \left(\sum_{i=1}^{N} C_{pu} + C_{pu} \times e^{\left(-\frac{\lambda_i}{\lambda_{i-1}}\right)} \right) = \sum_{k=1}^{h} \left(\sum_{i=1}^{N} C_{pu} \left(1 + e^{\left(-\frac{\lambda_i}{\lambda_{i-1}}\right)} \right) \right) \tag{23}$$

$\forall$ i $\in$ 1..N, $\beta > 1$ and $\varepsilon > 0$, we have an increasing production cost $C_{p,i}$. We recall that the production cost of the i^{th} batch is given by the following equation:

$$C_{p,i} = C_{pu} + C_{pu} \times e^{\left(-\frac{\lambda_i}{\lambda_{i-1}}\right)} \tag{24}$$

Thus,

$\forall i > 1$, $\beta > 1$, $\varepsilon > 0$, the production cost of the i^{th} batch ($C_{p,i}$) is a strictly increasing function. Using equations (4), (5), (8), (10), (17), (20), (21) and (23), the expected total profit per time unit corresponding to the proposed integrated policy (P T[N, h]) is given by:

$$PT[N,h] = \frac{\sum_{k=1}^{h} N \times e^{-\rho\times N} \times P_{\max} - \left(M_c \times Nb_{fk} + \sum_{i=1}^{N} \left(C_{pu} \times \left(1 + e^{\left(-\frac{\lambda_i}{\lambda_{i-1}}\right)} \right) \right) \right)}{\sum_{k=1}^{h} N \times Proc + \mu_c \times Nb_{fk} + \sum_{k=2}^{h} \mu_p \times e^{(k-2)\times\alpha_1} + \mu_p'} - \frac{\sum_{i=1}^{N} C_{rku} \times \left(\frac{\lambda_i}{\lambda_{N+1}} \right) + C_{rwu} \times e^{(h-1)\times\theta} + \sum_{k=2}^{h} M_p' \times e^{(k-2)\times\alpha_1}}{\sum_{k=1}^{h} N \times Proc + \mu_c \times Nb_{fk} + \sum_{k=2}^{h} \mu_p \times e^{(k-2)\times\alpha_1} + \mu_p'} \tag{25}$$

With

$$C_{rku} \times \left(\frac{\lambda_i}{\lambda_{N+1}} \right) + C_{pu} \times \left(1 + e^{\left(-\frac{\lambda_i}{\lambda_{i-1}}\right)} \right) \leq e^{-\rho\times i} \times P_{\max} \; \forall i \in 1..N$$

And

$$M_p' \times e^{(k-2)\times\alpha_1} \leq C_{rwu} \times e^{(h-1)\times\theta} \; \forall k \in 2..h$$

The objective is to determine, simultaneously, the optimal number of batches(N^*) produced before scheduling PM actions, and when the overhaul must be undertaken (h^*) maximizing the total profit per time unit (PT [N, h]).

4 NUMERICAL STUDY

Extensive numerical tests have been performed in order to test the consistency and robustness of the developed model. The goal is to find the optimal integrated strategy (N^*, h^*) for any given set of input parameters. Some of the obtained numerical results are presented in this section. Numerical data, considered for illustrative purpose, are summarized in Table 1 (mu stands for 'monetary units' and tu for 'time units'). MATHEMATICA software has been used to perform calculations and obtain the optimal solution (N^*, h^*) for any instance of the problem. It is important to mention that the parameters of the Weibull and Exponential distributions are chosen arbitrarily while respecting reliability constraints (i.e. increasing failure rate...). In practice, such distributions along with their parameters are obtained either through reliability tests or using historical data about the time to failure of the system and repair times. Several commercial statistics software can easily be used for this purpose.

Tableau 1. Numerical Data

System time to failure	F(t): Weibull distribution Shape parameter β = 2 Scale parameter = 100

Average time to failure = 88.6 tu	
Repair time	G(t): Exponential distribution Parameter η = 0.06
μ_c(tu)	16.67
P_{max}(mu)	1700
M_p(mu)	100
$M'_{p,1}$(mu)	400
M_c(mu)	2000
C_{rw}(mu)	2500
Proc(tu)	3
C_{pu}(mu)	350
C_{rku}(mu)	750
μ_p(tu)	10
μ'_p(tu)	25
α_1	0.1
α_2	0.15
θ	0.05
ρ	0.005

4.1 Numerical results

Maximizing the total expected profit expressed by equation (25), the optimal solution is obtained (Table 2).

N*	h^*	PT^*
15	5	167.433

N^* = 15: the optimal number of batches to be produced before undertaking a PM action.
(h^* − 1) = 4 PM actions must be performed and then at the end of the subsequent PM sub-cycle, an overhaul has to be performed to restore the machine to the 'as good as new state'.

5 CONCLUSION

The proposed study deals with an integrated maintenance quality policy taking into account the interaction between system degradation, output product quality and production cost. In fact, we considered a manufacturing system composed of a single machine producing items with different quality levels. Reworking is possible for low-quality products (second-rate and non-conforming products).

The manufacturing system is subject to random failures with an increasing failure rate. In order to slow down the degradation of the system limiting its undesirable impacts on the output product quality, and on the production cost, an integrated policy combining imperfect PM actions, repairs and overhaul, has been proposed and detailed in this paper.

Two decision variables are considered: the number N of batches to be produced before performing a PM action, and the number h of PM sub-cycles at the end of which an overhaul must be performed.

A mathematical model has been developed expressing the total expected profit per time unit as a function of the two decision variables, taking into account the revenues from sold products and the costs related to production, maintenance and rework, as well as the machine's reliability and the probability distribution of the times to have it repaired following failures.

Extensive numerical testing has been undertaken to test the consistency and robustness of the model. Some of the obtained results have been presented and discussed.

Several extensions of this work are currently under consideration. The major one consists in extending the model to manufacturing systems composed of several machines producing different types of products. Other possible extensions of this work consist in incorporating logistics related important aspects like transportation, inventory and warehousing, and customer service. In fact, such a supply chain oriented approach represents a promising area for future research in the development of industrial integrated strategies.

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