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Simulation-optimization of an inventory control policy in a distribution multi-echelon system with returns

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***Résumé* - Cette étude porte sur un système de distribution multi-échelon avec retour inspirés de la chaîne d'approvisionnement en boucle fermée des bouteilles de gaz de Air Liquide. La structure de distribution multi-échelons et la complexité induite par les retours de bouteilles renforcent les difficultés de la gestion des stocks dans cette chaîne d'approvisionnement. Notre objectif est de trouver une politique optimale de gestion périodique des stocks globalement plutôt que localement optimisée dans chaque usine de remplissage comme c'est fait par l'entreprise. La malédiction de la dimensionnalité induite par les ventes perdues de demandes clients insatisfaites disqualifie la programmation stochastique pour résoudre notre problème. Par conséquent, nous utilisons une approche de simulation-optimisation. Notre analyse numérique comparant le coût total moyen des stocks obtenu par l'approche de simulation-optimisation par rapport à l'heuristique locale appliquée par l'entreprise montre que les écarts entre les coûts de chaque méthode augmentent lorsque la variance de la demande augmente. Cet écart atteint 11% lorsque la variance de la demande est élevée. Nous montrons également que les écarts entre les coûts totaux moyens des stocks de différentes méthodes augmentent lorsque la variance de la distribution des échanges inégaux de bouteilles avec les clients augmente. Il atteint 13% lorsque cette variance est élevée. Ces résultats illustrent le gain que nous pouvons attendre lorsque nous optimisons les politiques de gestion périodique des stocks globalement plutôt que localement dans un système de distribution à plusieurs échelons. Ils mettent l'accent sur les impacts sur le coût des stocks de la relation entre la demande satisfaite et les bouteilles retournées.**
***Mots clés* - Chaîne d'approvisionnement en boucle fermée, gestion multi-échelon des stocks, système de distribution, simulation-optimisation.**

***Abstract* - This study focuses on a multi-echelon distribution system with returns inspired by the closed-loop supply chain of Air Liquide gas cylinders. The multi-echelon distribution structure and the complexity brought by cylinder returns strengthen the inventory control difficulties. We aim to find the optimal periodic review inventory control policies globally rather than locally optimized in each filling plant as done by the company. The curse of dimensionality induced by lost sales of unsatisfied customer demands disqualifies stochastic programming approaches. Therefore, we use a simulation-optimization approach. Our numerical analysis comparing the average total inventory cost found by the simulation-optimization approach against the local heuristic applied by the company show that the gaps between the cost of each method increase when demand variance increases. This gap reaches 11% when the demand variance is higher. We also show that the average inventory costs increase when the unequal exchange cylinder distribution's variance increases. It reaches 13% when this variance is higher. Those results illustrate the gain we can expect when we optimize inventory control policies globally rather than locally in a multi-echelon distribution system. They emphasize impacts on the inventory cost of the relations between satisfied demand and returned cylinders.**
***Keywords* - Closed-loop supply chain, multi-level inventory control, distribution system, simulation-optimisation.**

1 INTRODUCTION

Reusable packaging represents a critical asset for companies (Breen, 2006), mainly when they are not interchangeable and cannot be replaced temporarily with generic low-cost alternatives. It is the case for Air Liquide company which is specialised in producing and distributing industrial gases and providing services related to these gases. These gases are used in various applications, particularly in healthcare and industry, but also by individuals. Air Liquide uses three distribution channels to deliver its products: pipelines for the continuous supply of products in gaseous form to heavy industry, cryogenic tankers for the supply of liquefied products in large quantities to manufacturers and hospitals, and gas cylinders for the supply of smaller volume to a wide range of customers like artisans. Air Liquide records $9,487$ Million Euros of annual revenue in 2021 for Industrial Merchant activities and uses 20 million cylinders in its closed-loop supply chain of packaged gas cylinders. It represents 65% of the supply chain's overall cost.

From the company's inventory manager's point of view, setting the inventory control policies in closed-loop systems of packaged gas cylinders is challenging. Indeed, company managers reported difficulties managing return flows representing operational challenges. How

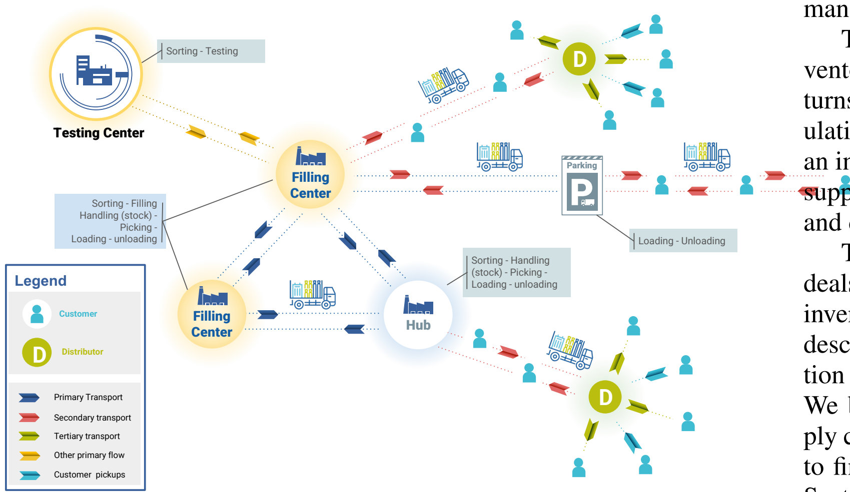

Figure 1: Air Liquide gas cylinder supply chain.

to determine the number of cylinders needed in the supply chain to ensure the quality of operations? How to reduce inventory in the supply chain by optimizing inventory policy control in an integrated manner instead of the local optimization used by the company? Indeed, each filling plant optimized its inventory control policy locally based on local demand and returned cylinders. They do not consider the multi-echelon distribution structure of the overall supply chain.

In this paper, we focus on the supply chain for the delivery of gas cylinders. Figure 1 shows the supply chain components: gas cylinder filling plants, distributors, hubs, and the testing center. The filling plants are factories where empty cylinders are returned to be sorted, refilled, and customers' orders are picked. The distributors are shops selling gas cylinders. Hubs are logistics platforms for storing and managing gas cylinder supplies. The testing center is a critical component necessary for repairing gas cylinders. Filling plants, Hubs, and distributors are directly in contact with customers and receive their daily orders. Therefore, they need an inventory of full gas cylinders to ensure the target service level. The specificity of this supply chain is that the filling plants must take back empty gas cylinders from customers to refill and meet future demand. It is therefore a closed-loop supply chain.

This supply chain consists of an upstream part, in which the testing center is the essential component, where faulty cylinders detected after sorting in the filling plants are repaired. In the downstream part, cylinders are sorted and filled; orders are prepared and delivered to customers by returning the empty cylinders needed for future fills. In this downstream part, distributors and hubs need inventory to meet customers' demands and not fill gas cylinders. The company controls the inventory of cylinders in the upstream and downstream parts of the supply chain separately. The inventory control of cylinders in the downstream part of the supply chain consists of setting replenishment policies to ensure customer service levels. In contrast, inventory control in the upstream part focuses on replacing defective cylinders in the downstream part without necessarily optimizing the level of service to customers. By doing so, the company optimizes its inventory control policy locally and in a decentralized way. We aim to integrate these two parts of the supply chain to jointly determine the best inventory control policy in a multi-echelon approach with cylinder returns.

At the customer ship out, we take back the empty cylinders and give full gas cylinders. If this exchange of cylinders with the customer is not guaranteed, we want to measure the impact on inventory needed to ensure service levels. Additionally, the unsatisfied demand of a given period is not back-ordered and is considered a lost sale.

The first contribution of this paper is the study of the performance of inventory control policies in a complex closed-loop supply chain. After building a discrete event simulation model of the supply chain, we study the impact of the relation between satisfied customer demands and returns on optimal inventory control policy.

The second contribution of the work is the optimization of the inventory control policy of multi-echelon distribution systems with returns and lost sales. We couple our closed-loop supply chain simulation model with an optimization module. Then, we optimize in an integrated manner the inventory control policies in the closed-loop supply chain. We compare our results against the results of the local and decentralised optimization method used by the company.

The remainder of this paper is organized as follows: Section 2 deals with a brief analysis of the relevant literature on multi-echelon inventory control with returns. Section 3 addresses demand modeling, describes our problem, and recalls some underlying assumptions. Section 4 focuses on modeling the problem and analyzing its dimension. We build a discrete event simulation model of the closed-loop supply chain. Section 5 presents a simulation-optimization-based method to find the optimal parameters for the echelon base stock policy. In Section 6, we apply the simulation-optimization approach to demand scenarios deduce from a real nominal product and compare the results with the company's actual solutions. We analyze the impact of the variability of the unequal exchange between the full cylinders delivered to the customer and the empty cylinders returned. In the last section, we conclude and give some perspectives on future works.

2 RELATED WORKS IN THE LITERATURE

The literature on closed-loop supply chains divides products into three categories (Gallego, 2010). Items for the Reusable Transport Items class are not in direct contact with the final product, Reusable Product Material class are in direct contact with the final product, and reusable products. Gas cylinders considered in this work are reusable transport materials.

The work of (DeCroix et al., 2005) addresses the problem of optimizing inventory control strategies in a multi-echelon serial system with returns. In this work, considering returns leads to a possible negative net demand. This work is an extension of the work of (Clark and Scarf, 1960), who pioneered the analysis of serial systems with non-negative demand. (DeCroix et al., 2005) state that the base-stock policy for a stationary problem with an infinite horizon is optimal under the assumption of approximating the relationship between the inventory position of a given level and the net inventory of the previous level. (DeCroix, 2006) is also interested in serial systems with remanufacturing. They show that in such a system when dumps excess inventory coming from customers at the most upstream level of the serial system, the optimal policy is one with three parameters. The first parameter is an order up-to-level inventory; the second is an order less-to-level inventory triggering the disposed of excess inventory, and the thirst parameter trigger recovery. When surplus cylinders are dumped at an intermediate level, determining the optimal inventory policy is more complex since it violates the most crucial property of serial systems. Namely, in a multi-echelon serial system, the only link between a level and its predecessor is the limitation of the inventory position by the net inventory of the predecessor level. The decision to dump a part of the inventory at an intermediate level of a multi-echelon serial system becomes a second link between the levels.

Several literature reviews address the optimization of inventory control policies in multi-echelon systems. One of the most recent is (Kok et al., 2018), which proposes a typology for classifying stochastic multi-echelon systems. The literature extensively studies multi-echelon serial systems, and several robust results allow us to determine the optimal inventory control policy in different cases. Multi-echelon assembly systems assimilate with serial multi-echelon systems and adopt his known results. For distribution systems, no results allow assimilation with the serial system, which remains a challenge.

Works on the distribution system proposed by (Zipkin., 2000) without returns and (Rong et al., 2017) focus on the case of demand following Poisson distribution. (Rong et al., 2017) studies two families of heuristics to find the optimal echelon base stock policy. The recursive scheme optimal for serial stationary multi-echelon serial systems (DeCroix et al., 2005) is adapted in a heuristic that provides good results.

Discrete event simulation widely models complex systems with stochastic parameters. It allows efficient evaluation of system performance against a set of pre-specified parameters. It is often coupled with a method that generates relevant scenarios based on the studied critical parameters. Several works in the literature on inventory control in the reverse supply chain use a discrete event simulation approach. It is notably the case with (Teunter et al., 2000), which uses simulations to evaluate inventory control policies based on inventory costs and product return rates. (Teunter and Vlachos, 2002) focuses on the case where returned products are remanufactured before they are used and evaluates the system's performance as a function of demand and return rate.

More advanced methods allow the generation of scenarios by intelligently exploring the space of experimental designs defined by the critical parameters to optimize specific system performance criteria. Meta-heuristic methods such as genetic algorithms often inspire optimization methods. (Li et al., 2009) use such an approach to optimize scheduling and inventory control policies in a system with remanufacturing. (Aras et al., 2006) coupled OpQuest with an arena-based simulation to determine the best parameters for inventory control in a system with remanufacturing. (Zolfagharinia et al., 2014) used a method that combines simulation and optimization to evaluate the target inventory levels of a periodic review policy in a supply chain where demand and returns vary over time and are modeled as a function of product lifetime. They developed a metaheuristic based on the genetic algorithm and compared it to OpQuest results.

Reviews of (Swisher et al., 2004) and (Fu et al., 2005) propose a classification and analysis of papers that use an approach that combines simulation and optimization.

In this work, we develop a discrete event simulation model built on Simul8 coupled with OpQuest to determine the best order up-to-level echelon inventories of the considered system. We study the case of a cylinder's gas supply chain of one testing center with two filling plants. This system is modeled as a multi-echelon distribution system with returns. Our problem integrates the complexity of a muti-echelon distribution system without the assumption of Poisson distribution of demand as in the works of (Rong et al., 2017) and (Zipkin., 2000). We consider returns in such a multi-echelon distribution system contrary to the work of (DeCroix et al., 2005) restricted to the serial multi-echelon system with returns. Contrary to the works of (DeCroix et al., 2005), which consider back-ordering of unsatisfied demand, all demand not fulfilled in his period is lost. Moreover, returns are correlated to the satisfied demand contrary to the independent assumptions between demand and returns made in the work of (DeCroix et al., 2005). With those features of our problem, it becomes tricky to use a stochastic programming approach as in the work of (DeCroix, 2006) because we cannot restrict the dimension of the state variables. Then, the stochastic programming approach will conduct to the curse of dimensionality.

3 PROBLEM STATEMENT

3.1 *Demand modeling*

Demand modeling is key in considering customers' demands and finding the best inventory control policies. This modeling depends on the consumption profile of the different products. In the literature, several classifications deal with the different product categories according to their demand profile. The well-known classification of (Syntetos et al., 2005) divides products into four categories: erratic, lumpy, smooth, and intermittent. We consider a classification close to this one and present products in four categories. Regular and irregular represent categories of products with high order frequency and low and high variability in order quantities, respectively. The sporadic and slow-mover categories represent products with low order frequency and more or less variability in order quantities. An extensive statistical study of the demand distribution for 481 products allows demand modeling for the different categories. It shows that the normal distribution is suitable for regular and irregular products. At the same time, the compound Poisson negative binomial distribution is robust enough to capture product demand for all categories, including sporadic and slow-moving products. This study focuses on stationary demand products modeled with normal distribution.

3.2 *Model assumptions*

This work aims to find the best inventory control policy for a distribution multi-echelon system of a single product with returns. The multi-echelon system represents a supply chain's gas cylinders consisting of a testing center and two filling plants. We consider the following assumptions:

- The planning horizon is finite,
- The demands for the different periods are independent of each other and identically distributed. Indeed, our extensive statistical analysis of demand for 481 products allows us to confirm this assumption.
- Unsatisfied demand at the end of the period is lost,
- We consider two inventory points in each filling plant, the inventory of full cylinders (finished product) and empty cylinders (semi-finished products). One inventory point at the testing center represents the inventory of repaired or externally supplied empty cylinders.
- Each inventory point has an unlimited capacity.
- Empty gas cylinders returned equal to full gas cylinders delivered plus an uncertainty representing the difference between the number of full gas cylinders shipped to customers and the number of gas cylinders returned. This uncertainty is a centered normal distribution.

3.3 *Problem description: Closed-loop supply chain of a filling center*

Figure 2 represents a testing center and two filling plants and illustrates the studied system. In each filling plant, the process is identical. The full cylinders inventory point holds the final product to meet customer demand. Order picking and deliveries are processed to meet customer demands. Empty cylinders collected from customer deliveries return to the filling plant for sorting and refilling. Each period, a portion of the defective or obsoleted cylinders are sorted out and sent to the testing center. After the sorting process, the second inventory point for empty cylinders receives the empty cylinders ready for filling.

Repairing empty cylinders in the testing center goes through several stages, mainly sorting, testing, painting, and assembly of new components. The testing process discharges some cylinders because they are too defective. An external supplier can also supply empty cylinders to the testing center.

We model the considered system as a multi-echelon distribution system with returns, as shown in figure 3.

The inventory point of full cylinders is stage 1. Stage 2 is the empty cylinders inventory point where returns from customers occur.

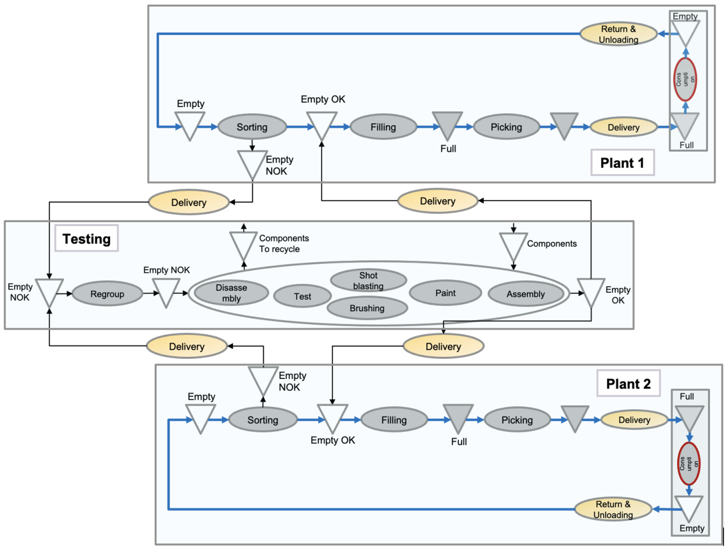

Figure 2: System with a testing center and two filling plants.

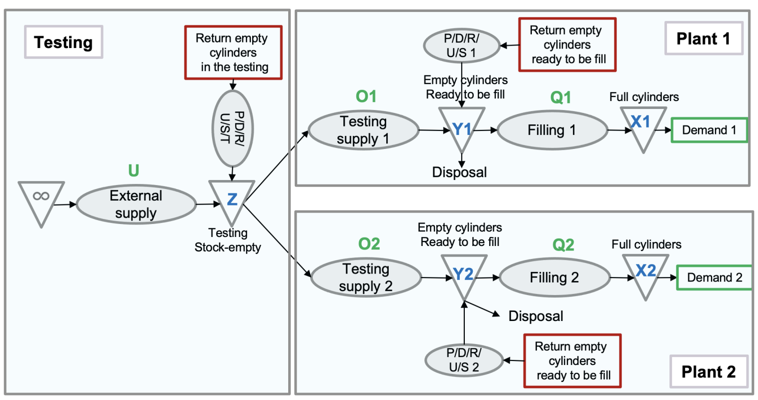

Figure 3: Distribution multi-echelon system with returns: P/D/R/U/S/T = Picking/ Delivery/ Return/ unloading /sorting/Testing and P/D/R/U/S = Picking/ Delivery/ Return/ Unloading /Sorting

Each plant has its own stage 1 and 2. Stage 3 is the inventory point at the testing center, and it is the predecessor of stage 2 in each filling plant. After the recovery process in the testing center, the inventory point at the testing center receives returned cylinders. That inventory point also receives new cylinders coming from an external supplier.

4 Model formulation and dimensional analysis

We use the following notations for the problem formulation:

- L_f, L_{rus} and L_{pd} are respectively the lead time of the filling, the lead time of the truck return, unloading and sorting of cylinders, and the lead time of the picking and delivery.
- L_t and L_a are the lead time of the testing and the lead time for an external supply of empty cylinders to the testing center.
- h_f, h_e and h_a are respectively the holding cost of full cylinders (stage 1), the holding cost of empty cylinders (stage 2), and the holding cost of empty cylinders in the testing center(stage 3).
- h^i_j are the echelon holding costs at stage $j = 1,2,3$. $h^i_1 = h_f$, $h^i_2 = h_e - h_f$, $h_3 = h_a - h_e$
- c_a is the unit cost per ordered cylinders to the testing.
- β is the targeted customer service level. We consider the fill rate to measure the customer service level. It is the ratio between the satisfied demand and the overall demand of customers.
- ξ: proportion of the discharged cylinders after the sorting. It is the proportion of cylinders not compliant with the filling and sent to the testing center after the sorting.
- T_t is the replenishment period of empty cylinders in the testing center,
- η: uncertainty on the exchanged cylinders. It represents the daily distribution of the gap between the delivered quantities and the number of empty cylinders returned by customers.
- $\mathbf{D}_i(t)$: demand of product at period t. We assume stationary demand, identical and independently distributed.

We consider the following sequence of events: At the beginning of each period, each inventory point receives the quantity previously ordered and the returned cylinders. We observe the inventory positions and decide to place new orders. The demand for the period occurs, and the net inventories at the end of the period allow us to determine the total inventory cost. In the inventory points for empty cylinders in filling plants, we disposed of some cylinders at the end of each period if there were too many cylinders compared to the required number. The external supplier receives his orders from the testing center only after each T_t period. Then, we want to find the optimal inventory control policies which allow deciding the order quantity to place in each plant.

4.1 *State variable and dimensional analysis*

The variables are:

- $\hat{\mathbf{X}}_i(t)$ and $\mathbf{X}_i(t)$ are respectively the echelon net inventory at stage 1 at time t in plant i and the echelon inventory-transit position at stage 1 at time t in plant i (full cylinders in plant i).
- $\mathbf{Y}_i(t)$ and $\mathbf{Y}_i(t)$ are, respectively, the echelon net inventory at stage 2 at time t in plant i and the echelon inventory-transit position at stage 2 at time t in plant i (overall cylinders in filling plant i).
- $\hat{\mathbf{Z}}(t)$ and $\mathbf{Z}(t)$ are respectively the echelon net inventory at stage 3 at time t and the echelon inventory-transit position at stage 3 at time t (overall cylinders).
- $\tilde{\mathbf{D}}_i(t)$ is the delivered cylinders at period t to the plant i'customers.
- $\mathbf{Q}_i(t)$ is the ordered quantity of full cylinders in plant i at the beginning of the period t.
- $\mathbf{O}_i(t)$ is the ordered quantity of empty cylinders in plant i at the beginning of the period t to the testing center.
- $\omega_i(t)$ is the disposed quantity of empty cylinders in plant i at the end of the period t.
- $\mathbf{U}(t)$ is the ordered quantity of new empty cylinders by the testing center at the beginning of the period t to an external supplier.

An inventory control policy specifies the quantity to order at all times. Note Π_d the set periodic review policies. When the π policy is implemented and the initial stock is (x_i, y_i, z), note $v(\pi,(x_i,y_i,z))$ the average expected cost over the horizon $[0,H]$ We then have: $v(\pi,(x_i,y_i,z)) = \frac{1}{H}.\mathbb{E}\Big[\sum_{t=0}^{H-1} h_3\hat{\mathbf{Z}}(t) + c_a\mathbf{U}(t) + \sum_i \Big[h^i_1\hat{\mathbf{X}}_i(t) + h^i_2\hat{\mathbf{Y}}_i(t)\Big]|\pi,(X_i(0),Y_i(0),Z(0)) = (x_i,y_i,z)\Big]$

The objective is to find the optimal policy π^* which minimizes the average expected cost, among the set of dynamic periodic review policies:

$$v^*(x_i,y_i,z) = \min_{\pi\in\Pi_d} v(\pi,(x_i,y_i,z))$$

The considered system is also subjected to some constraints. The lost sales assumption impacts the inventory dynamic of full cylinders in each filling plant. The inventory position of full cylinders (echelon 1) in each filling plant is always positive. The second harder constraint is related to the satisfaction of customer service level. The fill rate service level equal to the satisfied demand divided by overall demand on the horizon must be greater or equal to β. The other constraints are the classical constraints in multi-echelon inventory systems bounding each echelon's inventory position by its predecessor echelon's net inventory.

The state of the system a period t is $(\hat{x}_i(t), s_{i,1}(t), \hat{y}_i(t), \hat{z}(t), s_{i,2}(t), s_3(t), \tilde{d}_i(t))$ where $s_{i,1}(t) = (q_i(t-\tau))_{\tau\in\{1\ldots L_f-1\}}$, $s_{i,2}(t) = (o_i(t-\tau))_{\tau\in\{0\ldots L_t-1\}}$, $s_3(t) = (u(t-\tau))_{\tau\in\{0\ldots L_a-1\}}$ and $\tilde{d}_i(t) = (\tilde{d}_i(t-\tau))_{\tau\in\{0\ldots L_{pd}+L_{rus}+L_t-1\}}$. The state dimension is $2L_f + 2L_t + L_a + 2(L_{pd} + L_{rus} + L_t) + 3$, and it is too difficult to reduce due to the lost-sale assumption. So it becomes complicated to solve this problem with stochastic programming approaches. We will then use the simulation optimization technique.

4.2 *Discrete event simulation model of the distribution system*

Discrete event simulation commonly allows the modeling of complex systems that may contain stochastic parameters. It is suitable for systems where a list of events that occur at specific points in time describes the dynamics of state variables. The scheme in Figure 2 describes the flow of cylinders between filling plants and the testing centers.

The parameters for inventory, ordering costs, and the lead times of the different activities are deterministic input parameters of the simulation model.

The leading output performance indicators of the system are the customer service rate at each plant (IFR) and the average total inventory cost. The IFR is the ratio between the quantity of demand satisfied over the horizon and the total quantity demanded by customers. The simulation model aims to evaluate an inventory control policy for the multi-echelon distribution system under consideration.

We used Simul8 software to create the model. Due to the "stochastic" inputs, collecting stable and unbiased results requires the adjustment of several parameters. The warm-up period is one of those parameters. It is the period at the beginning of the simulation used to reduce the initialization bias of the system state variables. To determine this parameter, we used the method of (White, 1997) to choose the duration that minimizes the amplitude of the confidence interval of a performance measurement of the system. The warm-up period is taken as the duration at the beginning of the simulations to minimize the standard error of indicator's measures.

$$warm\ up = arg \min_{0\leq d<n} \sum_{i=d+1}^{n} \left[\frac{Y_i - \bar{Y}_n(j)}{n-d}\right]^2$$

where n the numbers of observations of the performance measure, Y_i is the ith observed measure and $\bar{Y}_n$ is the mean of all the observations.

The warm-up time is zero by choosing the average total inventory cost as the performance measure (see Figure 4). It means that our initialization assumption setting the echelon inventory with the echelon up-to-level inventory has no impact on the system's performance.

The second parameter to determine is the duration of results' collection necessary for stable results. Several experiments of duration allowed us to observe that a simulation horizon from 1000 units of time units gives stable performance measures, in particular, customer service rate and the average inventory cost (see figure 4 and 6). The simulation model allows us to measure several performance indicators. We mainly track the Echelon inventory position (see Figure 5), describing the system's state and allowing us to determine the number of cylinders to order depending on the inventory control policy used. We implemented rules to evaluate the performance of the echelon base-stock policies. Thus, as input parameters, we have $s_{i,1}$, $s_{i,2}$ with $i = 1,2$ and s_3 representing the echelon order up-to-level inventory of the echelon in plant i and in the testing center. For given values of these parameters, we can evaluate the customer service rate of each filling plant, the lost sales (see Figure 6), and the average total inventory cost (see Figure 4).

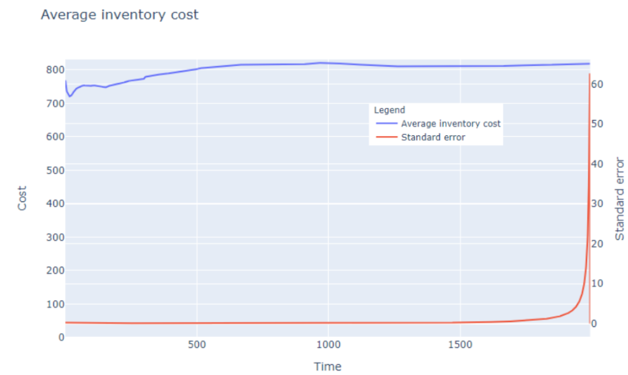

Figure 4: Warm up period analysis: average inventory cost and standard error regarding warm up period

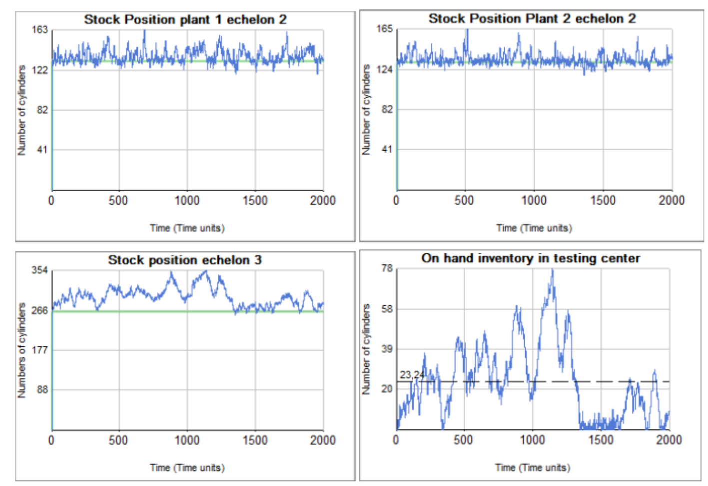

Figure 5: Echelon inventory position and on hand inventory at the testing center

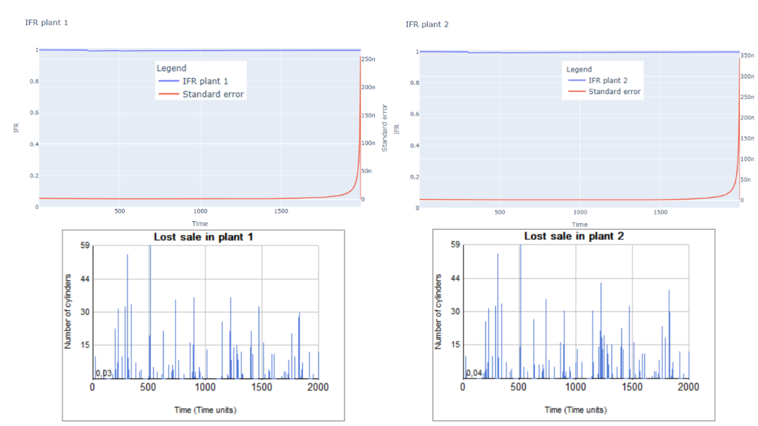

Figure 6: Lost sale and customer service level in the filling plants

To validate our simulation model, we performed a sensitivity analysis of the critical stochastic parameters, customer demand, and the distribution of unequally exchanged cylinders η. We examined the impact of varying more or less 10% of these stochastic parameters on the performance indicators. As shown in the table 1, the sensitivity of a slight variation in the distribution of unequally exchanged cylinders on the customer's service rate and average inventory cost is less than 1%. The sensitivity of demand to average inventory cost is also less than 1%, and it is more significant at customer service levels (2.68%). Indeed, increasing demand without changing the echelon order up-to-level inventory leads to decreased service rates.

The following section uses the simulation model to determine the optimal echelon base order up to level inventories to minimize the average total inventory costs while maintaining a target service level.

Table 1: Sensitivity analysis of demand and unequal exchange' cylinders

	Demand sensitivity analysis			
	Base scenario	scenario 1 (Base -10%)	scenario 2 (Base + 10%)	Sensitivity
std_exchange	2	2	2	
IFR_plant_1	99,76%	99,88%	97,58%	2,31%
IFR_plant_2	99,74%	99,91%	97,05%	2,86%
Average inventory cost	800,63	763,61	756,27	0,96%

	Exchange sensitivity analysis			
	Base scenario	scenario 1 (Base -10%)	scenario 2 (Base + 10%)	Sensitivity
std_exchange	2	1,8	2,2	
IFR_plant_1	99,92%	99,92%	99,93%	0,01%
IFR_plant_2	99,91%	99,89%	99,89%	0,00%
Average inventory cost	705,70	703,05	709,97	0,98%

5 A HYBRID SIMULATION OPTIMIZATION BASED METHOD

The echelon base stock policy is optimal for multi-echelon serial systems, as (Clark and Scarf, 1960) shows. Furthermore, the company uses a periodical review inventory control policy that is easy to implement. The review period is given in the company, and the problem is restricted to calculating the optimal up-to-level inventory to satisfy service level constraints. Therefore, we will determine the optimal up-to-level inventory of an echelon base stock policy for our multi-echelon distribution system with returns. Let denote by $s^*_{i,1}$, $s^*_{i,2}$, s^*_3 the optimal parameters of the echelon base stock policy of the various filling plants and testing centers. First, we determine the up-to-level

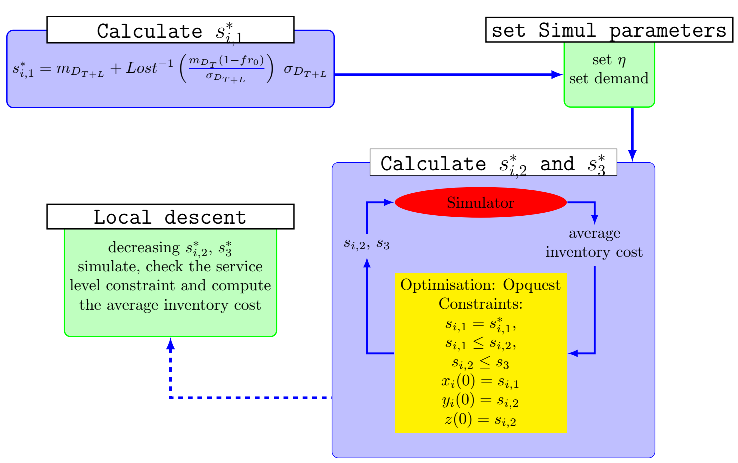

Figure 7: Methodology to calculate echelon base stock up-to- level parameters.

inventory of the first echelon of each filling plant $s^*_{i,1}$. Accordingly, we compute the safety stock in the case of a periodic control policy with one unit of the period' review. We use the fill rate $\beta = 98\%$ as a targeted service level. Then, we set the simulation parameters, particularly the demand and the variance of the distribution η representing the unequally exchanged cylinders. We are coupling the simulation model built on Simul8 with the OpQuest optimization engine to determine the best echelon base stock of stages 2 and 3. In the optimization module, we fix that initial inventories are equal to the echelon up-to-level inventories. Then, we add constraints ensuring that the order up-to-level echelon inventory of a given stage is always lower than the order up-to-level echelon inventory of its predecessor ($s^*_{i,1} \leq s^*_{i,2}$, $s^*_{i,2} \leq s^*_3$). With Opquest, we found the best up-to-level inventories for echelons 2 and 3 by coupling simulation and optimization; then, we ended with a local descent to reduce the average total inventory cost as much as possible.

6 RESULTS AND DISCUSSION

We consider a heuristic implemented in the company calculating the inventory target of each plant separately. Each plant is represented as a two-echelon serial system. Each two-echelon serial system is then decomposed into two single-echelon systems. The first one corresponds to echelon 1 with full cylinders alone, assuming empty cylinders are always available. The second system corresponds to echelon 2 and is defined as a single echelon system where a distribution of unequal exchange of cylinders with customers modifies the demand. The lead time of this second system is the cumulated lead time of all the activities on the observable part of the closed loop of each plant.

In the company, each filling plant receives cylinders sent to the testing center after a duration of L_t. During the reparation at the testing center, a proportion γ (5%) of cylinders is scrapped. Periodically, for each T_t times unit (60 times unit), each plant places an external order to complete the echelon 2 inventory position at the up-to-level of echelon 2.

The testing center centralizes the order placed by all plants and places a unique command to the external supplier. When the testing center receives the command, he sends to each plant the quantity they ordered. In actual practice, the testing center does not hold inventory. He repairs cylinders, sends them back immediately after, centralizes external orders placed by plants, and sends them directly to plants when orders arrive.

We apply our methodology to a nominal product with demand following a normal distribution, and the distribution η has normal centered distribution. The lead time parameters are $L_f = 1$ *time unit*, $L_{rus} = 1$ *time unit*, $L_{pd} = 1$ *time unit*, $L_t = 15$ *time units* and $L_a = 20$ *time units*. The echelon holding cost are $h^i_1 = 3$, $h^i_2 = 1$, $h_3 = 1$. The unit cost of external ordered cylinders to the testing is $c_a = 10$. The targeted customer service level is $\beta = 98\%$. The horizon planning is $H = 2000$ *time units*.

To determine order up to level parameters of echelon 2 and 3, we allow 15 min of OpQuest's running time for each demand and unequal exchange scenario.

Table 2 presents results given by our approach coupling simulation and optimization and compares them against solutions proposed by actual heuristics of the company. IFR_1 and IFR_2 are the service rate of each filling plant calculated by simulation. *Cost_heur* (*cost_sim*) is the average total inventory cost of the policy found with company heuristic (simulation optimization approach), and *gap* is equal to $\frac{(cost\_sim - Cost\_heur)}{Cost\_heur}$.

In the numerical comparison, we consider 6 demand scenarios deduced from a real nominal product with a regular demand profile following a normal distribution with a mean m_D equal to 60.94. To define each considered demand scenario, we vary the variance parameter σ_D between 0, 4, 8, 12, 16, and 20. For each demand scenario we consider different cylinder returns scenarios by varying the parameters σ_η between 0, 2, 4, 6, 8, and 10. Customers returned the same amount of cylinders they received in the scenario with σ_η equal to 0. Under the assumptions of equivalent exchange ($\sigma_\eta = 0$), the average total inventory cost of the centralized inventory control policy is always lower than that of the decentralized inventory policy applied in the company. The gap between those average total inventory cost increase when variability (σ_D) of the demand increase (see Figure 8). That equals 2.12% when demand is constant and reaches 11.8% when demand variance is 20. We observe the same trend when customer gas cylinder returns are not equivalent to the quantity delivered ($\sigma_\eta > 0$). In a centralized and decentralized inventory policy control, the average total inventory cost increase when the demand variant increase. The gaps between those inventories' average total inventory cost increase when demand variance increases (see figure 9).

When the demand of each filling plant is constant (demand scenario 1), the average total inventory cost increases when the variance

Table 2: Comparison of results of actual heuristic against solution find results founded with simulation optimization method.

				Actual heuristic						Simulation-optimization								
Demand scenario	m_D	σ_D	σ_η	$S_{1,1}, S_{2,1}$	$S_{1,2}, S_{2,2}$	**new**	IFR_1	IFR_2	**cost_heur**	$S_{1,1}, S_{2,1}$	$S_{1,2}$	$S_{2,2}$	S_3	**new**	IFR_1	IFR_2	**cost_Sim**	**gap(%)**
scenario 1	60.94	0	0	122	274	87	0.999	0.999	1397.14	122	258	258	548	126	0.998	0.998	1367.57	-2.12
			2	122	277	58	0.98	0.98	1434.73	122	258	258	548	153	0.996	0.996	1369.9	-4.52
			4	122	285	32	0.998	0.997	1550.18	122	260	259	635	255	0.997	0.997	1476.73	-4.74
			6	122	293	164	0.995	0.997	1561.02	122	258	261	634	302	0.995	0.997	1491.19	-4.47
			8	122	302	397	0.973	0.988	1611.25	122	260	259	635	359	0.994	0.994	1522.58	-5.5
			10	122	312	123	0.997	0.992	1765.09	122	272	279	579	36	0.998	0.998	1570.73	-11.01
scenario 2	60.94	4	0	125	285	93	0.997	0.997	1455.64	125	277	268	588	134	0.997	0.996	1449.54	-0.42
			2	125	287	105	0.996	0.996	1483.84	125	272	285	568	166	0.995	0.996	1456.27	-1.86
			4	125	291	49	0.997	0.994	1555.57	125	260	263	635	154	0.995	0.995	1496.97	-3.77
			6	125	299	151	0.98	0.98	1594.43	125	293	268	605	133	0.996	0.995	1528.2	-4.15
			8	125	307	90	0.989	0.995	1680.68	125	293	268	605	68	0.997	0.995	1569.56	-6.61
			10	125	316	223	0.995	0.987	1793.14	125	276	274	625	116	0.996	0.995	1615.85	-9.89
scenario 3	60.94	8	0	132	302	90	0.996	0.996	1544.45	132	277	268	588	128	0.996	0.994	1453.81	-5.87
			2	132	304	110	0.996	0.996	1565.85	132	276	274	625	204	0.996	0.995	1509.24	-3.62
			4	132	307	25	0.991	0.996	1662.31	132	276	274	625	123	0.995	0.995	1522.53	-8.41
			6	132	312	78	0.975	0.996	1757.31	132	309	295	542	28	0.995	0.995	1585.88	-9.76
			8	132	319	29	0.996	0.949	1760.19	132	306	316	640	193	0.995	0.996	1662.21	-5.57
			10	132	327	29	0.995	0.948	1926.63	132	306	316	640	89	0.995	0.996	1700.58	-11.73
scenario 4	60.94	12	0	141	322	94	0.996	0.996	1649.45	141	295	297	608	108	0.996	0.996	1554.86	-5.73
			2	141	324	117	0.98	0.98	1672.79	141	300	287	620	159	0.996	0.994	1555.32	-7.02
			4	141	326	212	0.98	0.98	1692.67	141	308	300	633	130	0.996	0.995	1617.48	-4.44
			6	141	330	83	0.994	0.995	1839.11	141	310	297	640	178	0.995	0.994	1629.76	-11.38
			8	141	336	198	0.994	0.989	1846.34	141	310	281	715	225	0.996	0.994	1700.96	-7.87
			10	141	342	43	0.996	0.847	2056.99	141	323	281	412	0	0.996	0.994	1706.18	-17.05
scenario 5	60.94	16	0	150	344	95	0.995	0.995	1763.79	150	300	299	626	105	0.995	0.995	1590.31	-9.84
			2	150	345	104	0.995	0.995	1778.68	150	301	303	626	114	0.994	0.995	1603.25	-9.86
			4	150	347	148	0.995	0.994	1809.7	150	319	305	683	203	0.995	0.995	1693.11	-6.44
			6	150	351	192	0.991	0.995	1864.58	150	319	305	683	190	0.995	0.995	1715.36	-8
			8	150	355	363	0.989	0.994	1878.44	150	319	305	683	247	0.994	0.994	1745.31	-7.09
			10	150	360	184	0.99	0.989	2135.11	150	305	302	678	176	0.994	0.995	1752.62	-17.91
scenario 6	60.94	20	0	160	367	96	0.995	0.995	1883.77	160	309	314	650	121	0.994	0.994	1660.15	-11.87
			2	160	367	79	0.98	0.98	1901.69	160	320	314	642	40	0.995	0.994	1691.42	-11.06
			4	160	369	105	0.995	0.995	1950.99	160	339	314	642	58	0.995	0.994	1713.36	-12.18
			6	160	372	137	0.995	0.995	1963.71	160	327	333	693	143	0.995	0.995	1803.55	-8.16
			8	160	376	9	0.995	0.98	2149.73	160	309	325	595	16	0.994	0.994	1825.29	-15.09
			10	160	381	171	0.994	0.995	2174.38	160	293	359	670	47	0.992	0.995	1883.03	-13.4

of unequal exchange cylinder distribution (σ_η) increases in the decentralized and centralized inventory control policy. The average total inventory cost decreases by 11.01% when σ_η equals 10. We observed the same trend for all the other demand scenarios with random demand ($\sigma_D > 0$).

More generally, the inventory policy control of the distribution systems with returns conducts less inventory hold in filling plants' closed-loop than the decentralized inventory control policy proposed by the heuristic company. The order-up-to inventory of echelon 2 of each plant is always lower for inventory control policy given by simulation optimization than the solution of the decentralized heuristics of the company.

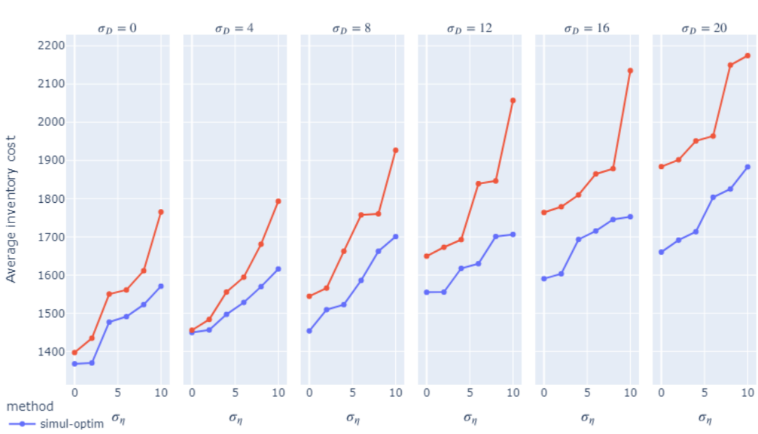

Figure 8: Comparison of the average inventory costs of the simulation optimization approach against company heuristic for each demand scenario

Our numerical results confirm the well-known results on inventory control about the inventory cost reductions when a multi-echelon inventory system is managed with a local inventory control policy rather than a globally optimized inventory policy. The fact that the inventory cost increase when demand variance increase is explained by the rise of safety stock needed to cover against that randomness demand. Our numerical results allowed us to analyze the impacts of unequal cylinder exchange with customers. The company commonly assumes that customers always return the same quantity of cylinders it receives during the delivery ($\sigma_\eta = 0$). As shown in Figure 8, this assumption significantly impacts the total inventory cost and underestimates the inventory needed to guarantee the target service level. The independent assumption of demand and returns is also commonly made in the inventory control with returns literature. In our study, returns depend on delivered quantities rather than the demand and are added with a random distribution of unequal exchange of cylinders with customers.

A multi-echelon distribution inventory system is known as difficult to control in the literature. Under a Poissonian assumption of demand, (Rong et al., 2017) proposed a recursive heuristic inspired by the optimal recursive scheme proposed by (DeCroix et al., 2005) for the serial multi-echelon system.

(Rong et al., 2017) proved that this recursive heuristic converges to the optimal solution. The returns are not considered in their study, and unsatisfied demand is assumed to be back-ordered. On another side, (DeCroix et al., 2005) proposed a recursive scheme to approximate the optimal solution of inventory control policies in a multi-echelon serial system with returns. In future work, we will propose a recursive heuristic inspired by the works of (Rong et al., 2017) and (DeCroix et al., 2005) for our distribution systems with returns.

The multi-echelon distribution system, the lost sale, and the depen-

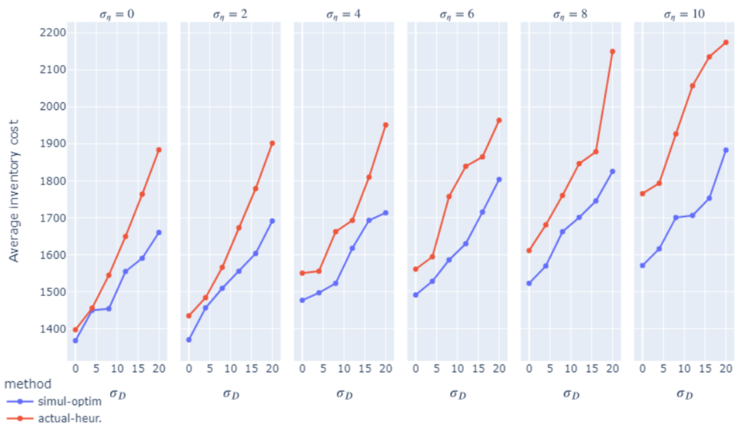

Figure 9: Comparison of the average inventory costs of the simulation optimization approach against company heuristic for each unequal cylinders exchange scenario (return scenario)

dency between returns and satisfied demand put together complexify our system and justify our choice of simulation optimization approach to optimize the inventory control policy globally rather than locally.

7 CONCLUSIONS AND PERSPECTIVES

We are interested in a multi-echelon distribution system with returns in this work. The aim is to find the optimal inventory control policy. After introducing the Air Liquide cylinder's supply chain that inspired this system, dimensional analysis of the problem led us toward methodology coupling simulation and optimization. We build a discrete event simulation model of the multi-echelon distribution system to evaluate an inventory control policy. We implement a method to determine the optimal up-to-level inventory. We perform a computational analysis of the results, emphasizing the impact of the variability of demand and unequal exchange of cylinders during deliveries on the average inventory cost.

Our numerical analysis comparing average total inventory cost found by simulation optimization approach against the local heuristic applied by the company show that the gaps between the cost of each method increase when demand variance increase (Table 3). This gap reaches 11% when the demand variance is higher. We also show that the average inventory costs increase when the unequal exchange cylinder distribution's variance increases (Table 3). It reaches 13% when this variance is higher.

Table 3: Mean of gaps between average total inventory costs of the centralized (simulation-optimization) and decentralized (apply by company) inventory control policies.

Mean over the 6 demand scenario (σ_D)		
m_D	σ_η	Mean cost_gap(%)
60.94	0	-5,98
60.94	2	-6,32
60.94	4	-6,66
60.94	6	-7,65
60.94	8	-7,96
60.94	10	-13,50

Mean over the 6 return scenarios (σ_η)			
Demands	m_D	σ_D	mean cost_gap(%)
scenario 1	60.94	0	-5,39
scenario 2	60.94	4	-4,45
scenario 3	60.94	8	-7,49
scenario 4	60.94	12	-8,92
scenario 5	60.94	16	-9,86
scenario 6	60.94	20	-11,96

Our study assumes that the overstocks generated by cylinder returns at echelon 2 of each filling plant are not disposed of. In practice, a transshipment of this overstock between plants allows fewer cylinder orders to the testing center or external supplier. In future work, we will study the inventory control policy with a new parameter representing the less-to-level inventory after what overstock will be disposed of.

Using the simulation-optimization approach to optimize the inventory control policy of our distribution system with returns is timeless. We will also investigate in the future work a tractable heuristic to find an optimal centralized periodical inventory control policy for our distribution system with returns.

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